Phys 131 2024

Lecture 32

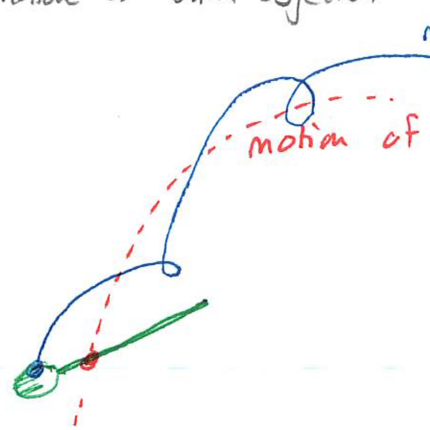
Weds: Review

Fri: Exam 3   Ch 9,10,11

Motion of "bulk" objects

Newton's laws apply strictly to point particles, although we have successfully used them on bulk objects. Any bulk object is a large collection of point particles. Can we modify Newton's laws to easily describe the motion of bulk objects?

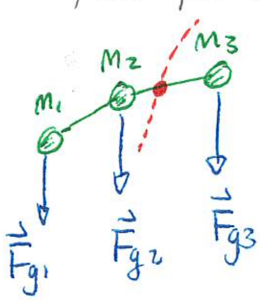
DEMO: UNSW videos



The video shows that, even with simple interactions (only gravity) the motion of any given point can be very complicated. However, there is a single special point whose motion is much simpler. This is called the center-of-mass.

Starting with Newton's Second and Third Laws one can show.

Bulk system (collection of point particles)



show external forces acting on the object

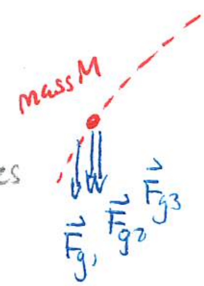
There is a special point called the center-of-mass whose motion satisfies

$$\vec{F} = M\vec{a}$$

where $\vec{F}$ = sum of all external forces

M = total mass

$\vec{a}$ = acceleration of c-o-m.



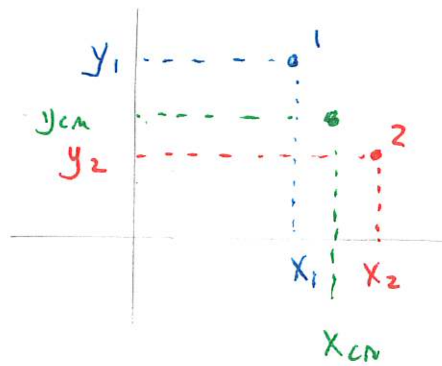
The center-of-mass behaves like a point particle with all external forces and all mass at the point particle.

↳ can partly assess the motion via an imaginary particle at the c-o-m.

The center of mass is a location, and one can show it is determined via:

For a collection of point particles:

* list particles via masses $m_1, m_2 \ldots$ and co-ordinates $(x_1, y_1), (x_2, y_2), \ldots$
* then the center-of-mass has co-ordinates (x_{cm}, y_{cm}) where



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\Sigma m_j x_j}{\Sigma m_i}$$

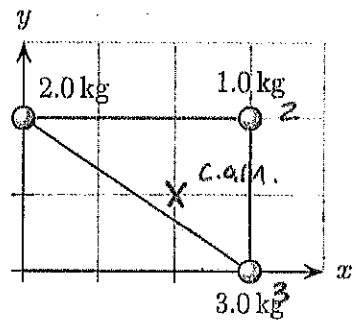
$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\Sigma m_j y_j}{\Sigma m_i}$$

The center-of-mass does not necessarily have to be located on the object.

~~Warm Up 1~~ after next exercise

274 Center of mass of three balls, 1

Three small balls are connected via massless rods in the illustrated configuration. Each grid block is 2.0 cm long. Determine the center of mass of the system. *(131Sp2023)*



Answer: List the data as follows

Object	mass	x	y
1	$m_1 = 2.0\,kg$	0.0 cm	4.0 cm
2	$m_2 = 1.0\,kg$	6.0 cm	4.0 cm
3	$m_3 = 3.0\,kg$	6.0 cm	0.0 cm

Then

$$\sum_j m_j = m_1 + m_2 + m_3 = 6.0\,kg$$

and

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_j} = \frac{1}{6.0\,kg}\left[0.0\,cm \times 2.0\,kg + 6.0\,cm \times 1.0\,kg + 6.0\,cm \times 3.0\,kg\right]$$

$$= \frac{1}{6.0\,kg}\,24\,cm\,kg = 4.0\,cm$$

$$y_{cm} = \frac{\sum y_i m_i}{\sum m_j} = \frac{1}{6.0\,kg}\left[4.0\,cm \times 2.0\,kg + 4.0\,cm \times 1.0\,kg + 0.0\,cm \times 3.0\,kg\right]$$

$$= 2.0\,cm$$

The location is illustrated.

For extended objects with a continuous mass distribution, one finds the center-of-mass via an integration process. If the object is symmetrical then the center-of-mass will be located at the symmetry axis.

Warm Up 1

Motion of the center-of-mass

We know that if we throw a ball, overall it will follow a parabolic trajectory associated with projectile motion. If however, the ball spins, any point on the surface will not follow a parabolic trajectory. Nevertheless the ball appears to undergo projectile motion. This is because we are implicitly following the center-of-mass. The motion of the center of mass is determined by

Add all forces on all parts of the system to give a net external force

$$\vec{F} = \sum_{\text{all parts of system}} \vec{F}_i$$

Acceleration of the c.o.m follows

$$\vec{F} = M\vec{a}$$

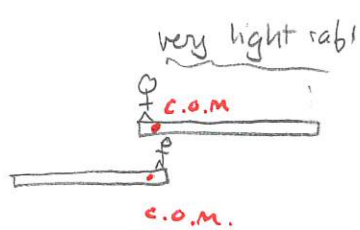
where M = total mass of system

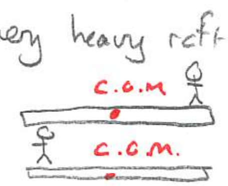
$\vec{a}$ = acceleration of c.o.m

Quiz 1 10% — 50% { 20% ~ 40%

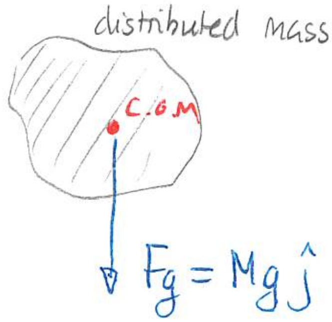
Quiz 2 90% {

In this case $\vec{F} = 0$ and $\vec{a} = 0$ for the c.o.m. Thus if the c.o.m is initially at rest it will remain at rest. How far the raft moves will depend on the relative masses.



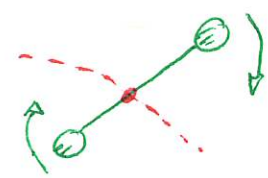


Note that, for an object in free-fall Earth's gravitational force effectively acts at the center-of-mass. We will use this in assessing bulk motion of objects.



Rotational and translational motion

For an object that rotates, we can often split the motion into two components:



~ translational motion of the center-of-mass

~ rotation of the parts about the center-of-mass.

We can treat these separately. To do this we need to develop the dynamics of rotational motion. We will ask

"How do forces affect the rotational state of motion."

This will be important for

1) rotating wheels, disks, ..
2) pivoting beams, rods, ...
3) rotating stars, planets, galaxies ..
4) atomic and molecular motion.