Mon: HW 5pm

Ex 317a, 318ab, 321, 322, 327, 329, 331, 333, 339

Tues: Warm Up 12

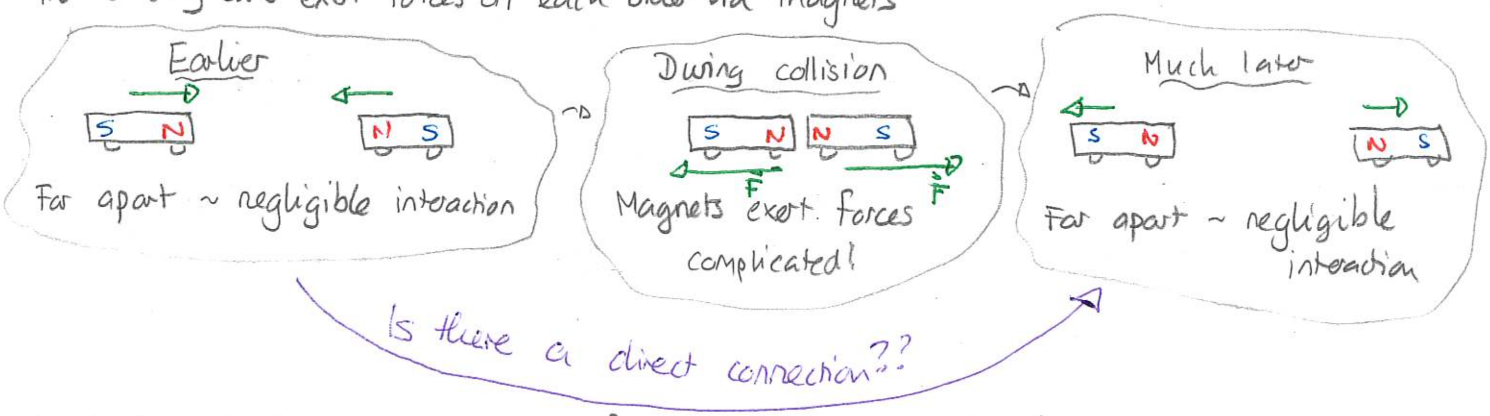
Weds: Review **Fri:** Exam 3 Energy + Momentum

Interacting isolated objects

Most physics situations include two or more objects that interact with each other but are otherwise effectively isolated from their surroundings.

DEMO: Track + carts colliding

The colliding carts exert forces on each other via magnets



Newton's mechanics promises that if we understand the forces (e.g. exerted by the magnets) then the entire motion of either cart can be predicted. However, we may only want to consider the motion long after the collision in relation to the motion long before. We might try to use energy but there could be non-conservative forces that do non-zero work. We will see that a new quantity, momentum, allows us to do the analysis.

DEMO: Ship collision video

DEMO: CERN proton-proton collision.

Momentum

Momentum combines mass and velocity via

> The momentum of an object with mass m and velocity $\vec{v}$ is
>
> $$\vec{p} = m\vec{v}$$
>
> units: kg m/s
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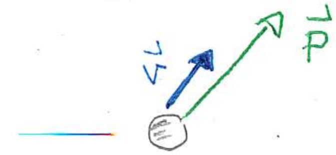
> 
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Note:

1) Momentum is a vector.
2) The direction of the momentum vector is the same as the direction of the velocity vector
3) Objects with the same velocity can have different momenta

Quiz 1 95% 90%

Momentum and Newton's Second Law

Momentum is directly connected to Newton's second law. Consider an object with mass m. Then

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}(m\vec{v}) \quad \text{provided mass is constant.}$$

$$\Rightarrow \boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$$

In many situations, this reformulation can be advantageous. Notice that, for a single isolated object, $\vec{F}_{net} = 0$ and this implies

$$\frac{d\vec{p}}{dt} = 0$$

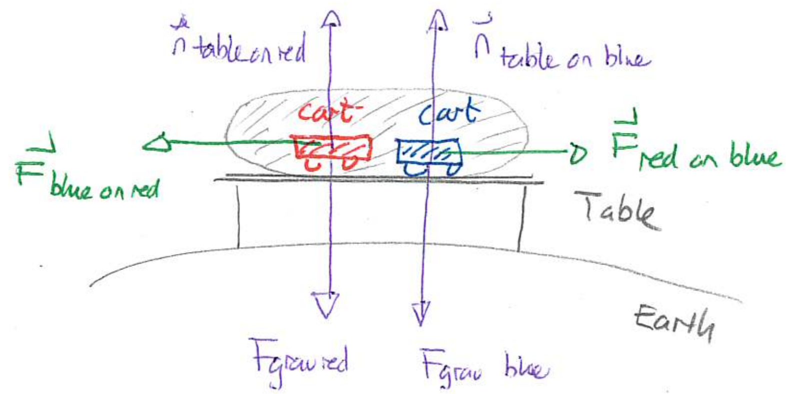
and in this case momentum would be constant.

Conservation of momentum

What happens with the momentum of a system of objects which is effectively isolated from its environment.

Consider two colliding carts on a table

We can divide the objects into



1) system (your choice!)

   e.g. red cart and blue cart

2) surroundings

   everything else : Earth, table, ...

We can divide the forces into:

1) internal ~ exerted by one part of the system on another

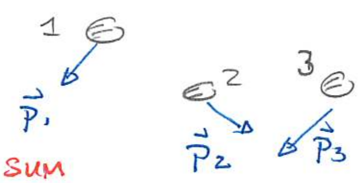
2) external ~ exerted by the surroundings on a part of the system.

In the illustrated example, the sum of the external forces is zero since the table is horizontal. They apparently do not affect the carts' motion. How does this affect the carts' momenta?

We first need:

The net (total) momentum of a system is

$$\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$



VECTOR SUM

Quiz 2 75% – 95% } 80% – 95%

Quiz 3 20% – 90% } 20% – 60%

Then Newton's Second and Third Laws give

Consider a system of objects. If the net external force on the system is zero then:

$$\frac{d\vec{p}_{tot}}{dt} = 0 \quad \Rightarrow \quad \text{total momentum is constant.}$$

Conservation of Momentum.

Proof: For a system of objects Newton's Second Law gives:

(Two objects)

$$\frac{d\vec{p}_{tot}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

But

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{net\ 1} = \underbrace{\vec{F}_{net\ ext\ 1}}_{\text{net external force on 1}} + \vec{F}_{2 on 1}$$

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{net\ ext\ 2} + \vec{F}_{1 on 2}$$

$$\Rightarrow \frac{d\vec{p}_{tot}}{dt} = \underbrace{\vec{F}_{net\ ext\ 1} + \vec{F}_{net\ ext\ 2}}_{\vec{F}_{ext\ net}} + \vec{F}_{2 on 1} + \vec{F}_{1 on 2}$$

But Newton's Third Law $\Rightarrow \vec{F}_{1 on 2} = -\vec{F}_{2 on 1}$. Thus.

$$\frac{d\vec{p}_{tot}}{dt} = \vec{F}_{ext\ net}$$

The result follows immediately. ∎

not done [Quiz 4

261 Space collision

A 100 kg astronaut is at rest in space. A 0.0050 kg fleck of paint moves toward the astronaut with speed 15×10^3 m/s. It collides with and sticks to the astronaut. *(131Sp2023)*

a) Write an expression for the total momentum of the system before the collision in terms of the masses and speeds of the astronaut and paint. Determine the total momentum of the system before the collision (assume that the paint fleck initially moves along the positive x axis).

b) Write an expression for the total momentum of the system after the collision in terms of the masses and speeds (after collision) of the astronaut and paint.

c) Use momentum conservation to determine the speed of the astronaut after the collision.

d) Now suppose that the paint fleck bounced off the astronaut and reverses direction with speed 8.0×10^3 m/s. Determine the speed of the astronaut after the collision.

Ans:

Before | After

We only need horizontal components of momentum

$v_{pi} = 15\times 10^3 \text{ m/s}$

$v_{ai} = 0 \text{ m/s}$

$v_{pf} = v_{af} = v_f = ?$

a) $p_{tot\,i} = m_p v_{pi} + m_a v_{ai} = 0.0050\text{kg} \times 15\times10^3 \text{ m/s} + 0 \text{ kgm/s} = 75 \text{ kgm/s}.$

b) $p_{tot\,f} = m_p v_{pf} + m_a v_{af} = m_p v_f + m_a v_f = (m_p + m_a) v_f$

c) $p_{tot\,f} = p_{tot\,i} \Rightarrow (m_p + m_a) v_f = 75 \text{ kgm/s}$

$\Rightarrow (100\text{kg} + 0.0050\text{kg}) v_f = 75 \text{ kgm/s}$

$\Rightarrow v_f = 0.75 \text{ m/s}$

d) $p_{tot\,f} = p_{tot\,i}$

$\Rightarrow m_p v_{pf} + m_a v_{af} = m_p v_{pi} + \cancel{m_a v_{ai}}$

$\Rightarrow m_p(-8.0\times10^3 \text{ m/s}) + m_a v_{af} = 75 \text{ kgm/s}$

$\Rightarrow -5.0\times10^{-3}\text{kg} \times 8.0\times10^3 \text{ m/s} + m_a v_{af} = 75 \text{ kgm/s}$

$\Rightarrow m_a v_{af} = 115 \text{ kgm/s}$

$\Rightarrow 100 \text{ kg}\, v_{af} = 115 \text{ kgm/s}$

$\Rightarrow v_{af} = 1.2 \text{ m/s.}$