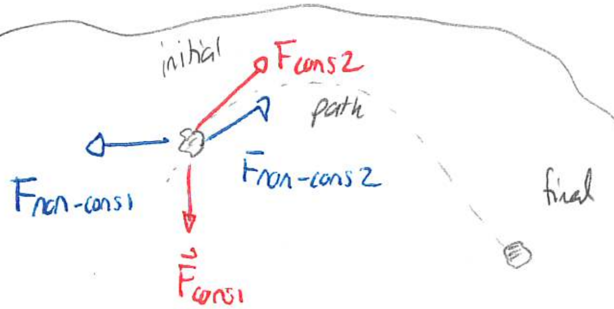
Mon: HW by 5pm

Ex: 317a, 318ab, 321, 322, 327, 329, 331, 333, 339

Energy conservation

The scheme for energy conservation is:



* Identify path between initial and final points
* Identify conservative forces
* Identify non-conservative forces

Total mechanical energy is

$$E = K + \underbrace{U_{cons1} + U_{cons2} + \dots}_{\text{one potential for each conservative force}}$$

Work done by non-conservative forces

$$W_{nc} = \underbrace{W_{non\text{-}cons1} + W_{non\text{-}cons2} + \dots}_{\text{one work for each non-conservative force}}$$

$$\Delta E = W_{nc}$$

OR

$$E_f = E_i + W_{nc}$$

Energy conservation is the case:

Conservation of Energy

If $W_{nc} = 0$ then $\Delta E = 0 \Rightarrow \Delta K + \Delta U_{cons1} + \Delta U_{cons2} + \dots = 0$

One can use this to think of energy as an exchange if $\Delta E = 0$ then

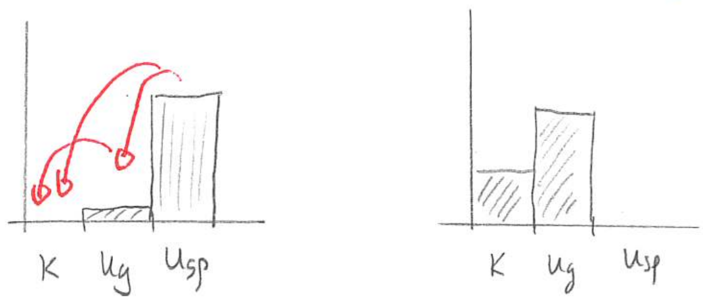
$$\Delta K + \Delta U_{cons1} + \Delta U_{cons2} = 0$$

and whenever one gains the other loses.

DEMO: PhET Springs / Masses

* Energy Tab

* No damping – suspend mass
- observe motion
- observe continuous energy exchange.



release from rest — later

* With damping – note thermal energy (apparently accounts for W_{nc})

This is analogous to moving money between bank accounts. The different energies are represented by different bank accounts.

Quiz 1 20% - 80% } 40% – 80%

Potential energy graphs and motion

One can plot potential energy and use the resulting graph to make inferences about the motion. Consider an example of the spring where $U_{sp} = \frac{1}{2}k(\Delta x)^2$. Choosing the origin of the co-ordinate system at the spring equilibrium point gives $\Delta x = x$

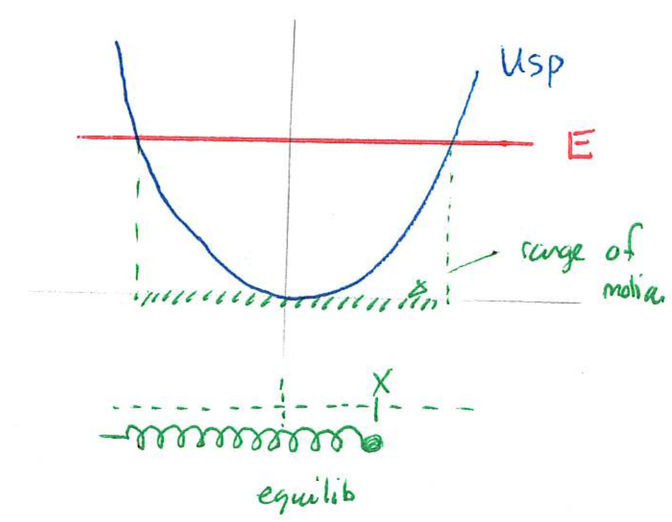
Thus

$$U_{sp} = \frac{1}{2}kx^2$$

Then letting $y=0$ gives $U_g = 0$

Thus

$$E = K + U_{sp}$$



Now suppose that the value of energy E is fixed. We require $K > 0$ and $K = E - U_{sp} \Rightarrow E > U_{sp}$. This restricts the range of motion

Quiz 2 70% – 90% } 30% – 60%

Quiz 3 80% – } 90%

These graphs can indicate:

1) range of motion: $\Rightarrow$ x so that $U < E$
2) turning points : $\Rightarrow$ x so that $U = E$
3) highest speed : $\Rightarrow$ x so that U is smallest.
4) equilibrium : $\Rightarrow$ x so that slope $U = 0$

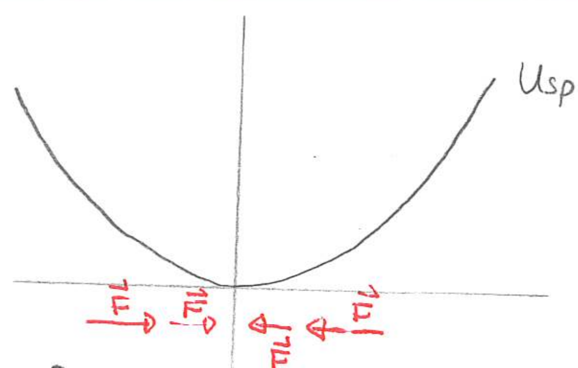
Slide 1
Slide 2
Slide 3

Force and potential energy

A very important feature of potentials is that they allow one to determine the associated force exactly. Again consider the object at the end of a spring. The forces point in the direction of decreasing potential. We can immediately see:

U has negative slope $\Rightarrow$ force right $\Rightarrow F_x > 0$
U has positive slope $\Rightarrow$ force left $\Rightarrow F_x < 0$

(F_x: x component of force)



One can show exactly that

> For any object moving along the x-axis under potential $U(x)$ the force associate with the potential is
>
> $$\vec{F} = F_x \hat{i}$$
>
> where
>
> $$F_x = -\frac{dU}{dx} = \text{negative slope of } U \text{ vs } x$$

Quiz 4 50% { 30% → 40%

Quiz 5 80%

324 Particle in a quadratic potential

A particle moves subject to an interaction described by the potential

$$U(x) = \frac{1}{2} kx^2 - bx$$

where $k = 140\,\mathrm{N/m}$ and $b = 35.0\,\mathrm{N}$. *(131F2024)*

* a) Determine any expression for the force associated with the potential.

b) Determine any locations where the force on the particle is zero. Is $U = 0\,\mathrm{J}$ at these locations?

c) Suppose that the particle is held at rest at $x = 0.0\,\mathrm{m}$. In which direction will it begin to move? Explain your answer.

Answer: a) $F_x = -\frac{dU}{dx} = -\left[\frac{1}{2}k2x - b\right]$

$\Rightarrow F_x = -kx + b \Rightarrow -140\,\mathrm{N/m}\,x + 35\,\mathrm{N} = F_x$

b) $F_x = 0 \Rightarrow 35\,\mathrm{N} = 140\,\mathrm{N/m}\,x$

$\Rightarrow x = \frac{35\,\mathrm{N}}{140\,\mathrm{N/m}} = 0.25\,\mathrm{m}$

The potential at this point is: $U = \frac{1}{2}\,140\,\mathrm{N/m}\left(\frac{1}{4}\mathrm{m}\right)^2 - 35\,\mathrm{N}\,\frac{1}{4}\mathrm{m}$

$= -4.375\,\mathrm{J}$ not zero.

c) $F_x = -140\,\mathrm{N/m}\;0\,\mathrm{m} + 35\,\mathrm{N} = 35\,\mathrm{N}$

$F_x \rightarrow$ $\Rightarrow$ will accelerate $\rightarrow$

$\Rightarrow$ will move $\rightarrow$