Tues: Discussion /quiz

Ex: 298, 299, 302, 304, 308, 310, 312, 313a

Energy conservation

The system for energy conservation, so far includes gravity and springs.

If gravity and springs are the only forces that do non-zero work on a system then the mechanical energy

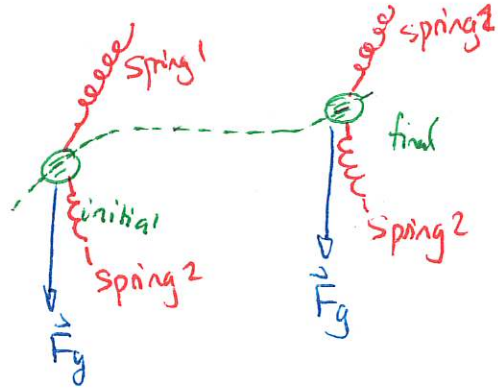
$$E = K + U_{grav} + U_{spring1} + U_{spring2}$$

where

$$K = \frac{1}{2}mv^2$$

$$U_{grav} = mgy$$

$$U_{spring} = \frac{1}{2}kx^2$$



Quiz: 100% ≷ 50%

DEMO:

In this situation there is no need to use work for any of these forces. We have

$$W_{grav} = -mgy_f + mgy_i \Rightarrow W_{grav} = -\Delta U_{grav}$$

$$W_{spring} = -\frac{1}{2}k(\Delta s_f)^2 + \frac{1}{2}k(\Delta s_i)^2 \Rightarrow W_{spring} = -\Delta U_{spring}$$

We then have

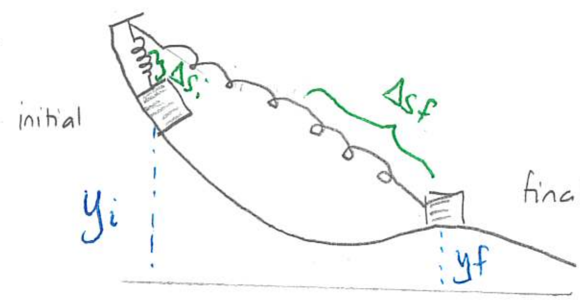
$$\Delta K = W_{net} = W_{grav} + W_{spring} \Rightarrow \Delta K = -\Delta U_{grav} - \Delta U_{spring}$$

$$\Rightarrow \Delta K + \Delta U_{grav} + \Delta U_{spring} = 0$$

$$\Rightarrow \Delta E = 0$$

Conservative forces

Consider the characteristics of the spring and gravity forces. For each of these forces



* there exists a potential energy U_{force} that only depends on the state of the system AND
* $W_{force} = -\Delta U_{force}$.

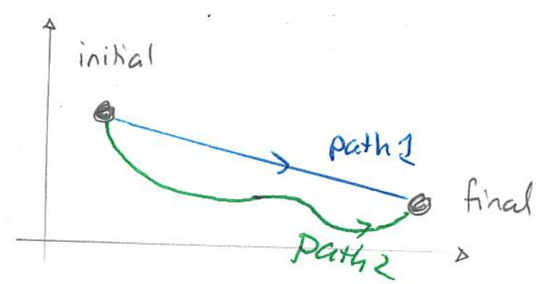
So

Force	Potential	Depends on state via
gravity	$U_g = mgy$	mass of object, vertical position
spring	$U_{sp} = \frac{1}{2}k(\Delta s)^2$	spring constant, spring stretch/compression

Do not depend on how object moves between initial and final locations

If $W_{force} = -\Delta U_{force}$

$$= -\left(U_{force\,f} - U_{force\,i}\right)$$



then we only need to know the initial and final locations to determine the work done. The path is irrelevant. So in the illustrated case

$$W_{grav}(\text{path } 1) = W_{grav}(\text{path } 2)$$

This will be true for any force that satisfies $W_{force} = -\Delta U_{force}$

Warm Up 1

Whenever the force has an associated potential energy then it is called a conservative force.

A force is <u>conservative</u> $\Longleftrightarrow$ The work done by the force only depends on the initial and final states of the system and not the path taken between these.

$\Updownarrow$ calculus in three dimensions

There exists a potential energy U_{force}, that only depends on the state of the system such that the work done by the force is

$$W_{force} = -\Delta U_{force}$$

Then two examples of conservative forces are

1) gravity (near Earth's surface)
2) spring forces.

We now consider this possibility for any other force

<u>Quiz 2</u> 40% – 80% ; 60%

<u>Quiz 3</u> 40% – 60% ; 80% – 70%

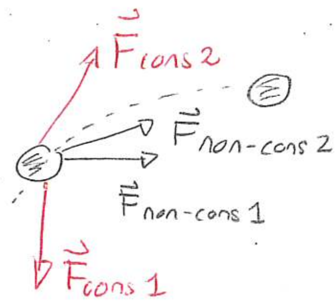
The last example shows that friction cannot be a conservative force. Thus there is no potential such that ~~$W_{friction} = -\Delta U_{friction}$~~. One can show that

The following are not conservative:

* friction $\Rightarrow$ no friction potential
* tension $\Rightarrow$ no tension potential
* normal force $\Rightarrow$ no normal potential

Mechanical energy in general

Can we still use $E = K + U_{grav} + \ldots$ if there are non-conservative forces that do work? We can by dividing the forces into conservative and non-conservative.



Then

$$\Delta K = W_{net}$$
$$= W_{cons\,1} + W_{cons\,2} + \ldots$$
$$+ W_{non\text{-}cons\,1} + W_{non\text{-}cons\,2} + \ldots$$

Then for conservative forces there is a potential s.t. $W_{cons\,j} = -\Delta U_{cons\,j}$.

So

$$\Delta K = -\Delta U_{cons\,1} - \Delta U_{cons\,2} - \Delta U_{cons\,3} \ldots + W_{non\text{-}cons\,1} + \ldots$$

$$\Rightarrow \Delta K + \Delta U_{cons\,1} + \Delta U_{cons\,2} + \ldots = W_{non\text{-}cons\,1} + W_{non\text{-}cons\,2} + \ldots$$

So we define the total energy as

$$E = K + U_{cons\,1} + U_{cons\,2} + \ldots$$

and the total work done by all non-conservative forces:

$$W_{nc} = W_{non\text{-}cons\,1} + W_{non\text{-}cons\,2} + \ldots$$

This gives

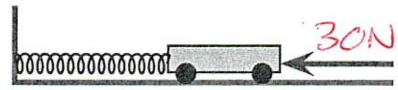
$$\boxed{\Delta E = W_{nc}}$$

Warm up2

315

~~248~~ Spring, cart and hand

A 5.0 kg cart can slide along a frictionless horizontal surface. A spring, with spring constant 400 N/m connects the cart to a wall. The spring is initially compressed by 0.25 m and the cart is held at rest. It is released and subsequently a hand pushes with a constant 30 N force against the spring as the spring relaxes. Determine the speed of the cart when the spring reaches its equilibrium position. *(131Sp2023)*



Ans:

Inital: spring released

Final: at equilibrium

$V_i = 0\,m/s$	$V_f = ?$
$X_i = 0.25\,m$	$X_f = 0$
$y_i = 0$	$y_f = 0$

$$\Delta E = W_{nc}$$

where

$$E = K + U_{grav} + U_{spring}$$

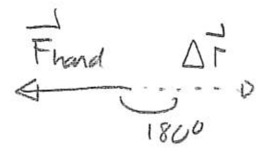
$$W_{nc} = W_{hand}$$

Then

$$W_{hand} = F_{hand}\,\Delta r \cos\theta$$

$$= 30N \times 0.25\,m \cos 180^\circ$$

$$= -7.5J$$



Now $\Delta E = W_{nc}$

$$\Rightarrow \quad E_f - E_i = W_{hand}$$

$$\Rightarrow \quad E_f = E_i + W_{hand}$$

$$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi} + W_{hand}$$

$$\frac{1}{2}mv_f^2 + \cancel{mgy_f} + \cancel{\frac{1}{2}kx_f^2} = \cancel{\frac{1}{2}mv_i^2} + \cancel{mgy_i} + \frac{1}{2}kx_i^2 - 7.5J$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2 - 7.5J$$

$$\frac{1}{2}\,5.0kg\,v_f^2 = \frac{1}{2}\,400N/m\,(0.25m)^2 - 7.5J$$

$$\Rightarrow \quad 2.5kg\,v_f^2 = 12.5J - 7.5J \quad \Rightarrow \quad v_f^2 = \frac{5.0J}{2.5kg} = 2\,m^2/s^2 \Rightarrow v_f = 1.4\,m/s$$