Thurs: Seminar WS 203 12:30

Fri: HW by 5pm

Ex 284, 285, 287, 288, 291, 293, ~~294~~, 295, 296

Energy conservation

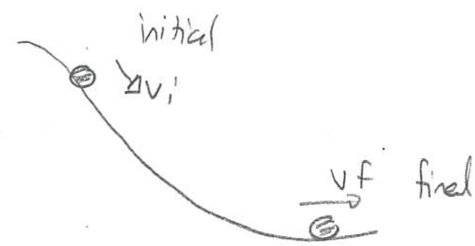
We were able to reformulate the work done by gravity giving a version of energy conservation:

If gravity is the only force that does non-zero work the

$$E = K + U_{grav}$$

stays constant. Here

$\underbrace{K = \frac{1}{2}mv^2}_{\text{kinetic energy}}$ and $\underbrace{U_{grav} = mgy}_{\text{gravitational potential energy.}}$



Alternative versions of this are:

$$\Delta E = 0 \iff \Delta K + \Delta U_{grav} = 0$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$E_f = E_i \iff K_f + U_{grav\,f} = K_i + U_{grav\,i}$$

Quiz 1 60% – 95% } 85%

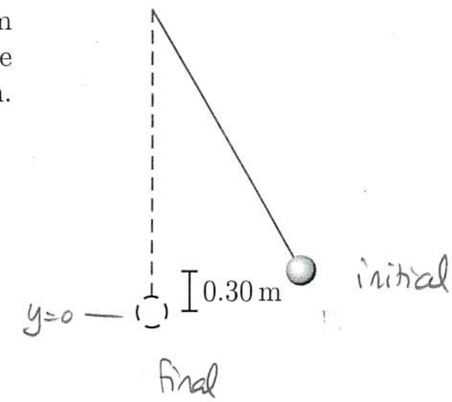
Quiz 2 100% } 60% – 90%

Quiz 3 60% ~ 70% }

DEMO: Loop-the-loop

294 Pendulum

A pendulum consists of a ball that swings from a string. A 1.4 m long pendulum is raised so that it is released from rest 0.30 m above its lowest point. Determine the maximum speed of the pendulum. *(131F2024)*



Gravity is the only force doing non-zero work

$E_f = E_i$

$\Rightarrow \quad K_f + U_{gf} = K_i + U_{gi}$

$v_i = 0\,m/s \qquad v_f = ??$

$y_i = 0.30\,m \qquad y_f = 0\,m$

$\Rightarrow \quad \frac{1}{2} m v_f^2 + mg\underbrace{\cancel{y_f}}_{0} = \frac{1}{2} m \underbrace{\cancel{v_i^2}}_{0} + mgy_i$

$\Rightarrow \quad \frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g y_i \quad \Rightarrow \quad v_f^2 = 2gy_i$

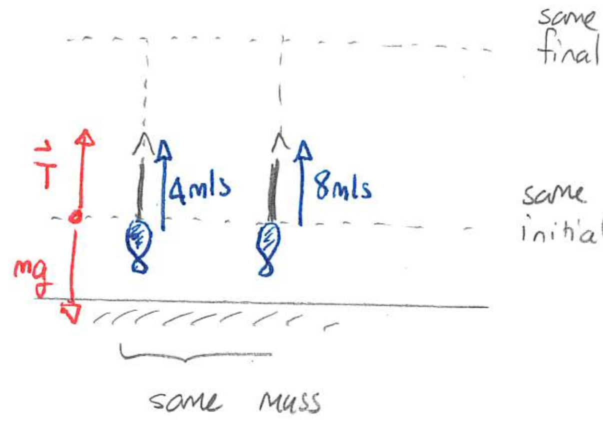
$\Rightarrow \quad v_f = \sqrt{2gy_i}$

$= \sqrt{2 \times 9.8\,m/s^2 \times 0.30\,m}$

$= \sqrt{5.88\,m^2/s^2} \Rightarrow$ $v_f = 2.43\,m/s$

Power:

Work and kinetic energy manage to ignore the time taken for a particular motion to unfold. Consider objects that move vertically at different constant speeds. In this case the tension in the lifting rope satisfies



$$T = mg \equiv \text{same for both.}$$

Thus the work done by the rope is the same for both. The change in kinetic energy will also be $\Delta K = 0$ for both.

The experience of lifting these will be different and this is reflected in the times taken. We then consider

Power ≈ rate at which work is done / energy is delivered

This is defined as:

Let W be the work done by a force along some trajectory. Let t be the time taken to traverse this trajectory. The power delivered by the force is

$$P = W/t$$

Units: Watts $W = J/s$

Warm Up 1 (from prev class)

Sometimes we determine power from a change in energy. Then

$$P = \frac{\Delta E}{\Delta t}$$ ← change in energy over time Δt.

Work done by a variable force in one dimension

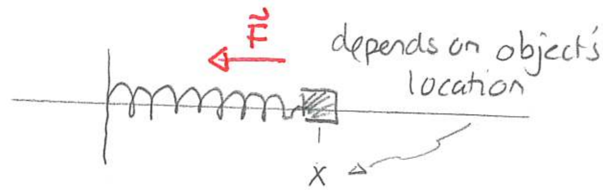
A simple situation of work done by a variable force is that where

* the object moves in a straight line
* the force only depends on the object's location

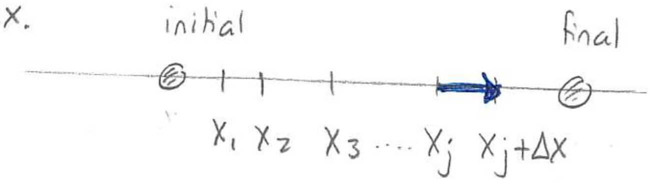
This is the case for a force exerted by a spring.

We will see that in this situation

$$\vec{F} = (ax + b)\,\hat{\imath}$$



where a, b are constants. We can break the motion into segments. Consider the segment from x_j to $x_j + \Delta x$. Then the force is



$$\vec{F} = \underbrace{F_x}_{x \text{ component}}\overbrace{(x)}^{\text{depends on } x}\,\hat{\imath}$$

and $\Delta\vec{r} = \Delta x\,\hat{\imath}$ $\Rightarrow$ work done for this segment is $\vec{F}\cdot\Delta\vec{r} = F_x(x)\Delta x$.

Thus the work done is

$$W = \sum_{\text{all segments}} F_x(x)\,\Delta x \longrightarrow W = \int F_x(x)\,dx$$

Spring forces

Springs are ubiquitous in physics. They produce forces that vary with position.

DEMO: ~~Spri~~ PhET Springs and Masses - variable springs

DEMO: PhET normal modes

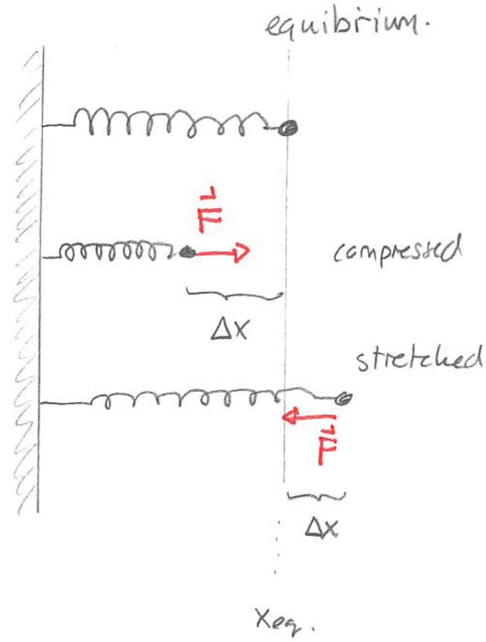
DEMO: Chem Tube 3D vibrations

The force law for springs involves:

1) any spring has an equilibrium position where it exerts zero force

2) any spring exerts a restoring force toward equilibrium.

3) the restoring force will depend on the stiffness of the particular spring



For a spring that pulls along the x direction the horizontal component of the spring force is

$$F_x = -k\,\Delta x$$

where $\Delta x = (x - x_{eq})$ is the displacement from spring equilibrium. This is Hooke's Law. The quantity k is called the spring constant (units N/m) and it depends on the particular spring.

We can show that:

If the spring moves an object attached to it from location x_i to x_f then the work done by the spring is

$$W_{spring} = -\tfrac{1}{2}k(\Delta x_f)^2 + \tfrac{1}{2}k(\Delta x_i)^2$$

$\Delta x_f = x_f - x_{eq}$

$\Delta x_i = x_i - x_{eq}$

Proof: $$W = \int_{x_i}^{x_f} F_x\,dx = -k\int_{x_i}^{x_f}(x - x_{eq})\,dx = -k\tfrac{1}{2}(x - x_{eq})^2\Big|_{x_i}^{x_f}$$

$$= -\tfrac{1}{2}k(x_f - x_{eq})^2 + \tfrac{1}{2}k(x_i - x_{eq})^2$$

$$= -\tfrac{1}{2}k(\Delta x_f)^2 + \tfrac{1}{2}k(\Delta x_i)^2$$

Quiz 4