Phys 131 2024

Lecture 25

Tues: Discussion /quiz

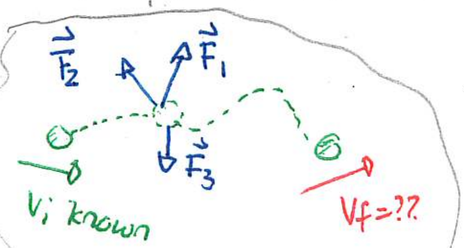
Ex 257, 260, 262, 263, 271, 275, 276, 279

Work and kinetic energy

Mathematical manipulations of Newton's 2nd Law result in the work-kinetic energy theorem.

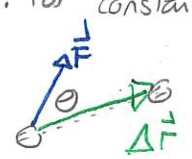
> Consider an object that moves from an initial to final location along a trajectory. Let
>
> W_{net} = net work done by all forces on the object
>
> Then
>
> $$W_{net} = \Delta K = K_{final} - K_{initial}.$$

An example of how this is used is:



Get work done by each force. For constant forces, straight lines

$$W = \vec{F}\cdot\Delta\vec{r}$$
$$= F\Delta r\cos\theta$$



Calculate $K_i = \frac{1}{2}mv_i^2$

Calculate $W_{net} = W_1 + W_2 + W_3 + \ldots$

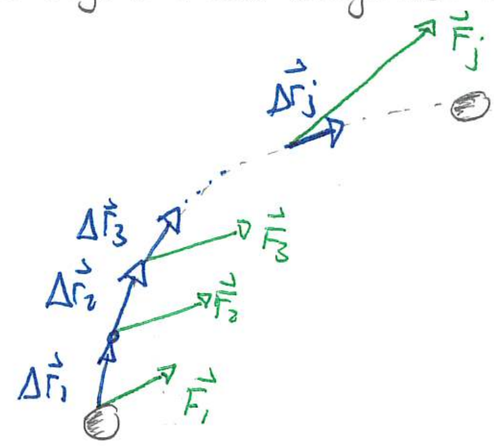
$K_f = K_i + W_{net}$ gives final KE

Get speed from $K_f = \frac{1}{2}mv_f^2$

Gives v_f

Work done when force is not constant and/or path is not straight

The definition of work so far applies only to straight line trajectories and constant forces. We need to extend this beyond these categories. To do this



* break the trajectory into small segments, each approximately straight
* the force is approximately constant on each segment
* the work done on the j^{th} segment is approximately

$$\vec{F}_j \cdot \Delta\vec{r}_j$$

* the total work done over the entire trajectory is the sum of these pieces

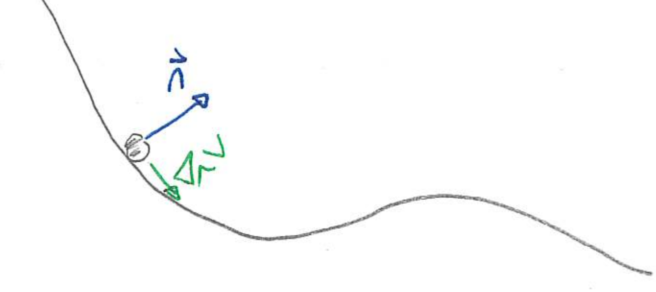
$$W \approx \vec{F}_1 \cdot \Delta\vec{r}_1 + \vec{F}_2 \cdot \Delta\vec{r}_2 + \dots + \vec{F}_j \cdot \Delta\vec{r}_j + \dots \qquad = \sum \vec{F}_j \cdot \Delta\vec{r}_j$$

As the segments become smaller and smaller (eventually $\to 0$ in length) the approximation becomes increasingly accurate. In the limit of infinitely many segments, each with length $\to 0$ we get an integral (technically a line integral)

Quiz 1 70% − 95% $\lessgtr$ 80% − 85%

Warm-Up 1

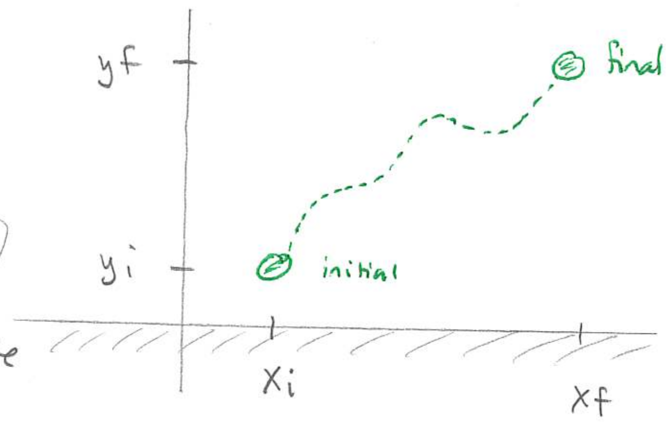
Note that $\vec{n}$ is always perpendicular to the instantaneous direction of motion. Thus $\vec{n} \cdot \Delta\vec{r} = 0$. Adding over all segments gives $W_{normal} = 0$



Work done by gravitational forces

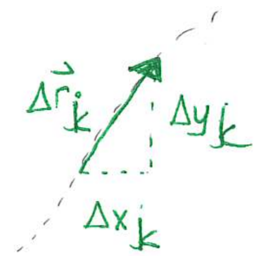
In situations where objects move near Earth's surface, we will need methods to compute the work done by Earth's gravitational force for any type of trajectory. We can show

> The work done by gravity (near Earth's surface) on an object with mass m is
>
> $$\boxed{W_{grav} = -mg\,\Delta y = -mg(y_f - y_i)}$$
>
> where y_i = initial vertical co-ordinate
> y_f = final " "



Quiz 2: 60% –

Proof: The curve trajectory can be broken into a succession of small straight trajectories. For the k^{th} segment



$$\Delta \vec{r}_k = \Delta x_k \hat{\imath} + \Delta y_k \hat{\jmath}$$

$$\vec{F}_k = 0\,\hat{\imath} - mg\,\hat{\jmath}$$

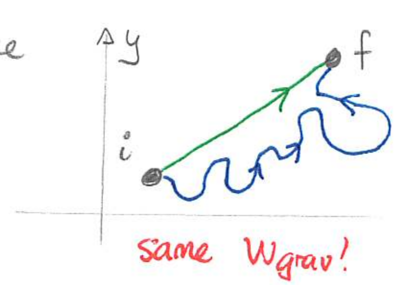
$$\Rightarrow \quad \vec{F}_k \cdot \Delta \vec{r}_k = -mg\,\Delta y_k$$

Then the work done is $W = \sum(-mg\Delta y_k) = -mg\sum_k \Delta y_k$

$$= -mg(y_f - y_i)$$

Note that:

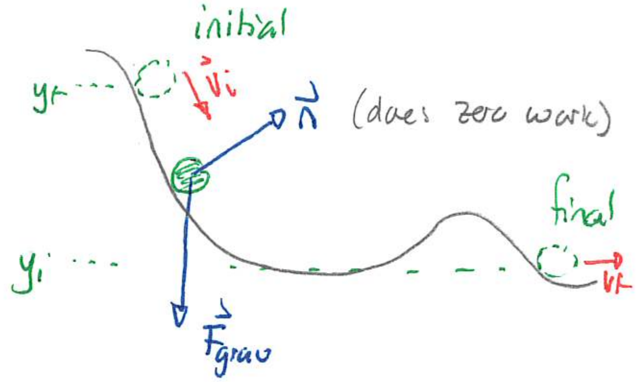
> The work done by gravity only depends on the initial and final locations/points and not the path between these



same W_{grav}!

Gravitational Potential Energy

Suppose that gravity is the only force that does non-zero work. We will now recast the work-kinetic energy theorem to provide a more convenient calculational approach.



$$\Delta K = W_{net} = W_{grav}$$

$$\Rightarrow K_f - K_i = -mg(y_f - y_i)$$

$$\Rightarrow \cancel{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2} \quad \Rightarrow K_f + mgy_f = K_i + mgy_i$$

Thus we define

> The gravitational potential energy of an object with mass m at vertical position y is
>
> $$U_{grav} = mgy.$$
>
> Units: J

Then we define

> The total (mechanical) energy of the system at any instant is
>
> $$E = K + U_{grav}$$

Thus for the situation:

> If gravity is the only force that does non-zero work on the system then the quantity
>
> $$E = K + U_{grav}$$
>
> is constant along the trajectory.
>
> Example of: The conservation of energy

We can apply this to any two points along the trajectory, giving

$$\boxed{E_f = E_i} \iff \boxed{K_f + U_{grav\,f} = K_i + U_{grav\,i}}$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$\boxed{\Delta E = 0} \iff \boxed{\Delta K + \Delta U_{grav} = 0} \iff \boxed{\Delta K = -\Delta U_{grav}}$$

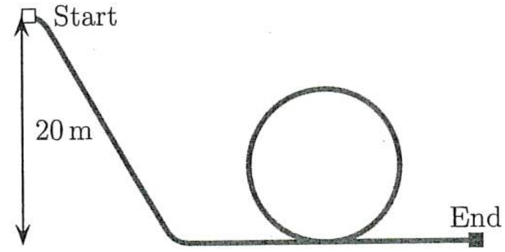
DEMO: PhET Energy Skate Park (Basics).

* Intro tab
* W track
* Energy bar graph.

After Exercise { Quiz 3, Quiz 4 }

290 ~~236~~ Loop-the-loop rollercoaster

A 100 kg rollercoaster starts from rest at the top of the illustrated track. The rollercoaster completes the loop. Determine speed of the rollercoaster at the end of the track. Ignore friction and air resistance. *(131Sp2023)*



Answer: Initial = start track.

Final = end track.

$y_i =$ $\qquad$ $y_f =$

$v_i =$ $\qquad$ $v_f =$

The normal force does zero work.

$$E_f = E_i$$

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_f^2 + \cancel{mgy_f}^{0} = \cancel{\frac{1}{2}mv_i^2}^{0} + mgy_i$$

$$\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}gy_i$$

$$\frac{1}{2}v_f^2 = gy_i$$

$$v_f^2 = 2gy_i$$

$$v_f = \sqrt{2gy_i}$$

$$= \sqrt{2 \times 9.8\,m/s^2 \times 20\,m}$$

$$= 20\,m/s$$

$$\Rightarrow \frac{1}{2}100\,kg\; v_f^2 = 100\,kg \times 9.8\,m/s^2 \times 20\,m$$

$$50\,kg\; v_f^2 = 1.96 \times 10^4\,J$$

$$v_f^2 = \frac{1.96 \times 10^4\,J}{50\,kg}$$

$$v_f^2 = 392\,m^2/s^2$$

$$v_f = \sqrt{392\,m^2/s^2}$$

$$= 20\,m/s$$