Phys 131 2024

Lecture 24

Weds: Review

Fri: Exam II Ch 5,6,7,8

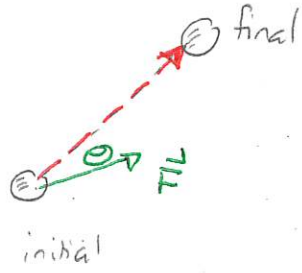
2022 ~~Midterm 2~~ Class exam 2 } All questions.

2023 Class Exam #2

Work

Recall that for a constant force acting on an object that moves in a straight line, the work done by the force is:

$$W = F \Delta r \cos\theta$$



where F = magnitude of force ≥ 0

Δr = distance traveled ≥ 0

θ = angle between $\vec{F}$ and $\Delta\vec{r}$

It appears that work is a form of product that involves the two vectors $\vec{F}$ and $\Delta\vec{r}$ as well the angle between these. This requires a type of vector product that produces a number.

Dot Product of Vectors

The "dot product" of two vectors is an operation producing a scalar number from a pair of vectors

Given two vectors

Feed into → Dot Product Machine → output → Single number denoted $\vec{A} \cdot \vec{B}$

The definition of the dot product is:

Given vectors $\vec{A}$, $\vec{B}$ express each in terms of the usual unit vectors

$$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$$

Then

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Quiz 1 80% ~ 100% } 80% ~ 95%

~~Warm Up 1~~ Quiz 2 95% } 95%

Quiz 3 95% } 75% - 95%

Warm Up 1

Notes: 1) $\vec{A} \cdot \vec{B}$ is a scalar and no unit vectors remain.

2) $\vec{A} \cdot \vec{B}$ can be positive, negative or zero.

One can prove, from the definition, that

1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

2) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

3) $\vec{A} \cdot (\alpha \vec{B}) = \alpha (\vec{A} \cdot \vec{B})$ for any number α

4) $\hat{\imath} \cdot \hat{\imath} = 1$ $\quad \hat{\imath} \cdot \hat{\jmath} = 0$

$\hat{\jmath} \cdot \hat{\jmath} = 1$ $\quad \hat{\jmath} \cdot \hat{k} = 0$

$\hat{k} \cdot \hat{k} = 1$ $\quad \hat{k} \cdot \hat{\imath} = 0$

A further derivation is that.

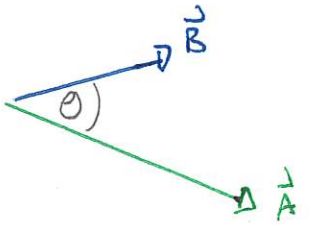
For any two vectors $\vec{A}, \vec{B}$,

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where A = magnitude of $\vec{A}$

B = " " $\vec{B}$

θ = angle from $\vec{A}$ to $\vec{B}$



Quiz 4 80% - 100% ≳ 80%

Note that, if $\vec{A} \cdot \vec{B} = 0$ then $\theta = 90°$. Thus

$\vec{A} \cdot \vec{B} = 0 \iff \vec{A}, \vec{B}$ are perpendicular.

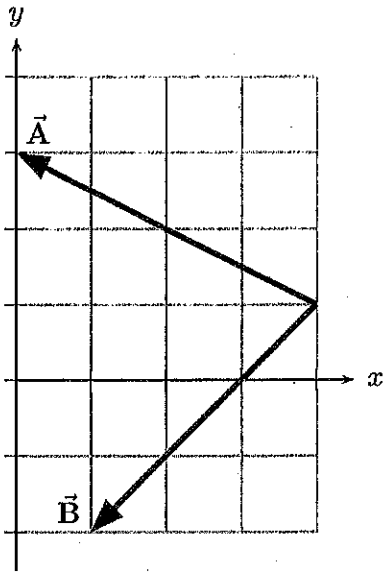
203 Dot product of two vectors, graphical, 2

Consider the vectors $\vec{A}$ and $\vec{B}$. *(131Sp2023)*

a) Determine the magnitude of each vector.

b) Determine the dot product of the vectors.

c) Use the dot product to determine the angle between the vectors.



Answer:

$\vec{A} = -4\hat{i} + 2\hat{j}$

$\vec{B} = -3\hat{i} - 3\hat{j}$

a) $A = \sqrt{A_x^2 + A_y^2} = \sqrt{4^2+2^2} = \sqrt{20} = 4.5$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{3^2+3^2} = \sqrt{18} = 4.2$

b) $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$

$= (-4)(-3) + (2)(-3) = 12 - 6 \Rightarrow \vec{A}\cdot\vec{B} = 6$

c) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$6 = \sqrt{20}\sqrt{18}\cos\theta \Rightarrow \cos\theta = \frac{6}{\sqrt{20}\sqrt{18}} = \frac{6}{\sqrt{4\cdot5}\sqrt{9\cdot2}} = \frac{1}{\sqrt{10}}$

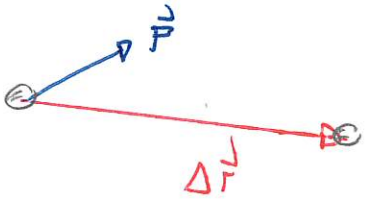
$= 0.316$

$\Rightarrow \theta = \cos^{-1}(0.316)$

$\Rightarrow \theta = 72^\circ$

Work in terms of dot product

We can combine the previous definition of work and the dot product to get:

> Suppose a constant force $\vec{F}$ acts on an object that moves in a straight line during an interval. Then if $\Delta\vec{r}$ is the displacement during the interval, the work done by the force is
>
> $$W = \vec{F} \cdot \Delta\vec{r}$$
>
> 
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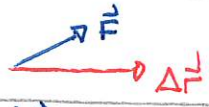
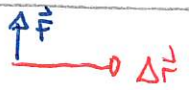
Warm Up 2

Note that:

Angle	Situation	W	Force tends to
$0 \leq \theta < 90°$		Positive	Speed up object
$\theta = 90°$		0	Does not affect speed
$90° < \theta \leq 180°$		Negative	Slow object.