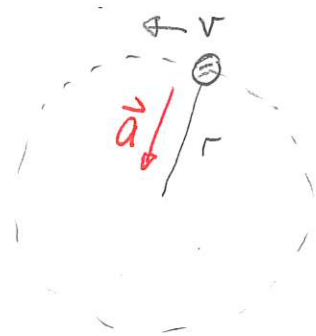
Thurs: Seminar 12:30 Wubben 203

Fri: Fall break

Mon: HW by 5pm

Ex: 235, 239, 245, 247, 250, 251, 252abc, 256

## Circular Motion




Recall that for circular motion, speed and angular velocity are related by

$$v = \omega r$$

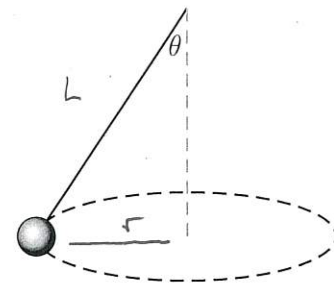
Then for uniform circular motion, the acceleration is radially inward with magnitude

$$a_c = v^2/r \qquad \text{or} \qquad a_c = \omega^2 r$$

Beyond this Newton's Second Law relates force to acceleration without any modification.

## 253 Conical pendulum

A ball with mass $m$ swings with a constant speed at the end of a string with length $L$. The angle between the string and the vertical, $\theta$, is constant. Suppose that one can easily measure $L, m$ and $\theta$. We would like to determine expressions for other quantities in terms of these. *(131F2024)*



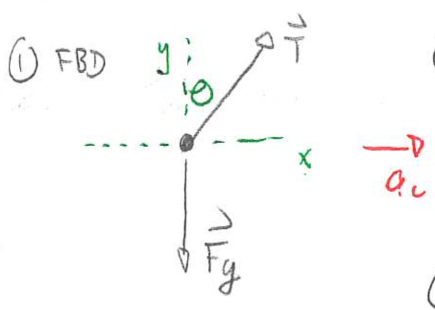

a) Determine an expression for the speed of the ball in terms of $m, \theta, L$ and constants. For a given string length and angle are there multiple possible speeds at which the ball can move? Are there multiple possible masses such that the ball can trace the same path for a given string length and angle?

b) Determine an expression for the angular velocity of the ball in terms of $m, \theta, L$ and constants.

c) Determine an expression for the period of orbit of the ball in terms of $m, \theta, L$ and constants.

Answer: a) Consider the situation when the ball is at the left

① FBD




② Newton's 2nd Law $\sum F_{ix} = ma_x \Rightarrow \sum F_{ix} = ma_c = mv^2/r$

$\sum F_{iy} = ma_y = 0$

③ Components $\sum F_{ix} = T\sin\theta \Rightarrow T\sin\theta = mv^2/r$

$\sum F_{iy} = 0 \Rightarrow T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg$

Then $r/L = \sin\theta \Rightarrow r = L\sin\theta$ gives

$$T\sin\theta = \frac{mv^2}{L\sin\theta} \qquad T\cos\theta = mg$$

$$\Rightarrow T = \frac{mv^2}{L\sin^2\theta} \qquad \Rightarrow T = \frac{mg}{\cos\theta}$$

combine $\frac{\cancel{m}v^2}{L\sin^2\theta} = \frac{\cancel{m}g}{\cos\theta} \Rightarrow v^2 = \frac{gL\sin^2\theta}{\cos\theta}$

$$\Rightarrow v = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}$$

Thus, for a given string length and angle there is only one speed possible. However, it would work for <u>any mass</u>

b) $$V = \omega r = \omega L \sin\theta$$

Thus $$V^2 = \omega^2 L^2 \cancel{\sin^2\theta} = g \frac{\cancel{L}\,\cancel{\sin^2\theta}}{\cos\theta}$$

$$\Rightarrow \omega^2 = \frac{g}{L\cos\theta} \Rightarrow \omega = \sqrt{\frac{g}{L\cos\theta}}$$

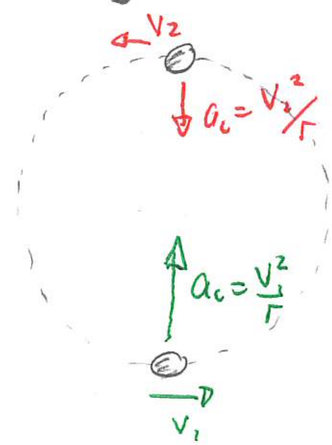
c) $$V = \frac{2\pi r}{t_p} \Rightarrow t_p = \frac{2\pi r}{V} = \frac{2\pi L\sin\theta}{\sqrt{g L \sin^2\theta/\cos\theta}}$$

$$= 2\pi\sqrt{\frac{L}{g}\cos\theta}$$

# Objects moving in vertical circles

When objects move in vertical circles the gravitational force, always pointing downward can result in the object speeding up + slowing down. The acceleration will not always be radially inward

However, if the motion is "symmetrical" about the highest and lowest points then at those points the acceleration is radially inward. Thus we find that:




At the top or bottom the acceleration is radially inward with

$$a = V^2/r = \omega^2 r$$

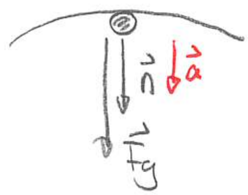
Quiz! 70% – 90%

40% – ~~70~~ 90%

## 248 Loop-the-loop

A 50 g ball slides, without rolling, inside a vertically oriented circular loop with radius 80 cm. In two different situations, the ball is set into motion so that its speed is i) 4.0 m/s and the top of the loop and ii) 4.0 m/s and the bottom of the loop. Determine the normal force exerted by the loop on the ball in each case. *(131F2024)*

Top:




$$\sum F_{iy} = ma_y$$

$$-n - mg = m\left(-\frac{v^2}{r}\right)$$

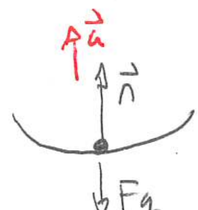
$$\Rightarrow \quad n + mg = mv^2/r$$

$$\Rightarrow \quad n = mv^2/r - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= 0.050\,\text{kg}\left(\frac{(4.0\,\text{m/s})^2}{0.80\,\text{m}} - 9.8\,\text{m/s}^2\right)$$

$$n = 0.51\,\text{N}$$

Bottom




$$\sum F_{iy} = ma_y$$

$$n - mg = mv^2/r \Rightarrow n = m\left(\frac{v^2}{r} + g\right)$$

$$\Rightarrow n = 0.050\,\text{kg}\left(\frac{(4.0\,\text{m/s})^2}{0.80\,\text{m}} + 9.8\,\text{m/s}^2\right)$$

$$\Rightarrow n = 1.5\,\text{N}$$

Quiz 2. One round 50%

Quiz 3