Phys 131 2024

Lecture 21

Tues: Discussion / Quiz

Ex: 233, 234, 238, 241, 242, 243, 255

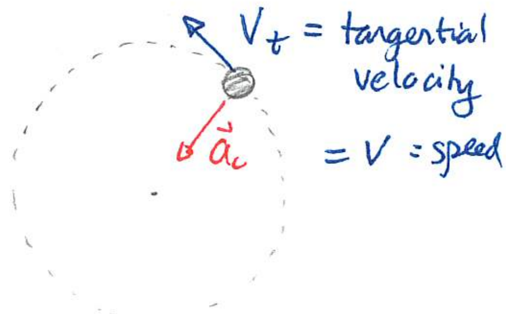
## Angular kinematic quantities

The angular velocity is the rate at which angular position, $\theta$, changes:

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

Then this is related to the speed or "tangential velocity" via

$$\boxed{v = \omega r}$$



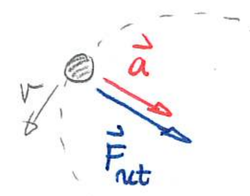

For uniform circular motion, the acceleration is radially inward with magnitude (centripetal acceleration)

$$a_c = \frac{v^2}{r}$$

Quiz! 30% — 60% ||| 50% - 70% $\Rightarrow$ $\boxed{a_c = \omega^2 r}$

## Dynamics of uniform circular motion

We know that, for uniform circular motion, there is an acceleration pointing radially inwards. Thus the net force is non-zero and there must be at least one force present.




We now use Newton's system of mechanics to relate forces to accelerations and motion for uniform circular motion.

## 237 Child swinging on ice

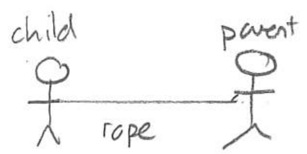
A parent and child are each on a horizontal sheet of ice. The parent is fixed to the ice and swings the child, who is connected by a horizontal rope to the parent. The child slides without any friction with constant velocity. *(131F2024)*

a) Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the speed of the child.

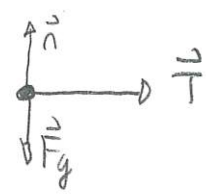
b) Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the period of motion of the child.

c) Determine an expression for period of motion of the child so that the tension felt by the parent is larger the force that the parent must exert to hold teh child at rest off the ground.

Answer: a) Horizontal side view




child FBD




child mass m

$\longrightarrow \vec{a}$

$a_c = v^2/r$

$$\sum F_{ix} = m a_x \Rightarrow T = m a_x$$

$$\Rightarrow T = mv^2/r \Rightarrow \boxed{T = mv^2/L}$$ where $L$ = length of rope

b) period = time for one circle $= t_p$. Then $v = \dfrac{2\pi L}{t_p}$

$$T = \frac{m(2\pi L/t_p)^2}{L} = \frac{m\,4\pi^2 L^2}{t_p^2 L} \Rightarrow \boxed{T = m\frac{4\pi^2 L}{t_p^2}}$$

c) The force require to hold the child at rest is $mg$. We need

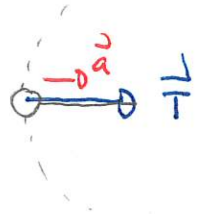
$$T > mg \Rightarrow \frac{\cancel{m}\,4\pi^2 L}{t_p^2} > \cancel{m}g \Rightarrow \frac{4\pi^2 L}{g} > t_p^2$$

$$\Rightarrow t_p < \sqrt{\frac{4\pi^2 L}{g}}$$

Check, if $L = 2.0\,m$ we need

$$t_p < \sqrt{\frac{4\pi^2 \times 2m}{9.8\,m/s^2}} = 2.8\,s$$

Note that the kinematics notion of acceleration now matches forces.



Consider a ball on a string. Assume that the ball swings on the end of the string in a horizontal circle. Then the tension points radially inward. This provides the net force and thus $\vec{F}_{net}$ is radially inward. Thus acceleration is radially inward.

**Quiz 2** 40% – 60% & 70% – 80%

Note that $T = ma_c \Rightarrow T = m\omega^2 r \Rightarrow \omega = \sqrt{\frac{T}{mr}}$

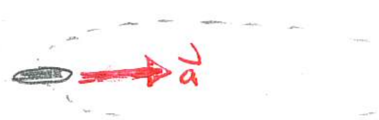
and this will answer the question. DEMO:

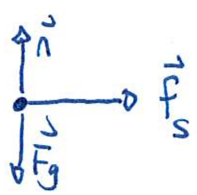
**Quiz 3** 10% – 40% & 50%

This analysis applies to forces other than tension, including friction.

**Warm Up 1**

side view





$\rightarrow$ radially inward acceleration

$\Rightarrow$ Net force radially inward

$\rightarrow$ must be an inward force $\Rightarrow$ friction!

It's static friction

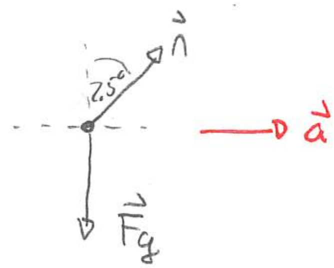
**Warm Up 2**

**254 Speed on a banked turn on a road**

A road is constructed with a turn of radius 600 m. The surface of the road is banked at an angle of 2.5° from the horizontal. Determine the speed such that a car can complete the turn without sliding sideways on the road. *(131F2024)*

Answer:


FBD




use axes s.t. accel is along x-axis. Then

$\sum F_{ix} = ma_x$

$\sum F_{iy} = ma_y = 0$

| | x | y |
|---|---|---|
| $\vec{F_g}$ | 0 | $-mg$ |
| $\vec{n}$ | $n\sin\theta$ | $n\cos\theta$ |

So $\sum F_{ix} = ma_x$

$\Rightarrow n\sin\theta = ma_x \qquad a_x = v^2/r$

$\Rightarrow n\sin\theta = mv^2/r \quad \sim (1)$

$\sum F_{iy} = 0 \Rightarrow n\cos\theta - mg = 0 \Rightarrow n = \dfrac{mg}{\cos\theta} \quad \sim (2)$

Combine (1) and (2)

$$\cancel{m}g\frac{\sin\theta}{\cos\theta} = \cancel{m}v^2/r$$

$$\Rightarrow gr\tan\theta = v^2 \Rightarrow v = \sqrt{gr\tan\theta}$$

$$= \sqrt{9.8\,m/s^2 \times 600\,m \times \tan 2.5^\circ}$$

$$= \sqrt{257\,m^2/s^2} \Rightarrow v = 16\,m/s$$



Quiz 4