Mon: Warm Up 8 (D2L)

Next HW: Mon 14 Oct.

Quiz 1 - Pose as a challenge to be answered in person

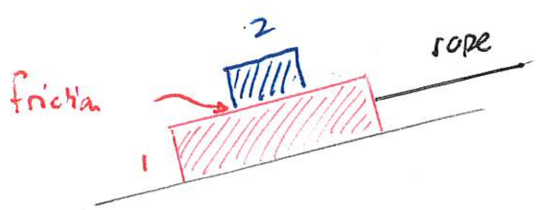
Interacting objects: different motions

We can have objects that interact and which do not share the same acceleration

~~Demo~~ DEMO: PhET Normal Modes

DEMO: Chem Tube 3D - Methane vibrations

A simpler situation involves objects that slip relative to each other. Again the strategy is:



Apply Newtons 2nd Law to object 1
↳ gives expressions involving $T, a_1, \ldots$

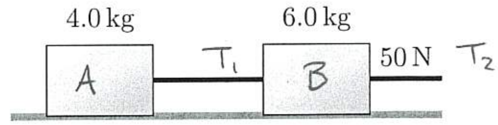
Apply Newton's 2nd Law to object 2
↳ gives expressions involving $a_2 \ldots$

Use Newton's Third Law to relate $\vec{F}_{2\,on\,1}$ and $\vec{F}_{1\,on\,2}$

→ Combine algebra

215 Connected objects: friction

Two boxes can move along a horizontal surface. There is no friction between the 6.0 kg box and the surface. There is friction for the other box: the coefficient of static friction is 0.70 and the coefficient of kinetic friction is 0.50. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N. *(131F2024)*

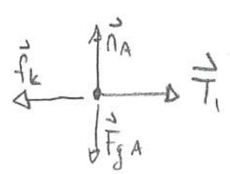


a) Determine the acceleration of each box.

b) Determine the tension in the rope connecting the boxes.

Answer: a)

Box A



$\sum F_{ix} = m_A a_x$; $\sum F_{iy} = 0$

$\Rightarrow T_1 - f_k = m_A a_x$; $\Rightarrow n_A - m_A g = 0$

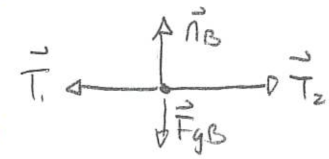
$\Rightarrow T_1 - \mu_k n_A = m_A a_x$; $\Rightarrow n_A = m_A g$

$T_1 - \mu_k m_A g = m_A a_x$

$T_1 = m_A a_x + \mu_k m_A g$

combine

Box B



$\sum F_{ix} = m_B a_x$; $\sum F_{iy} = 0$

$T_2 - T_1 = m_B a_x$; $\Rightarrow n_B - m_B g = 0$

$T_2 = T_1 + m_B a_x$; $n_B = m_B g$

$T_2 = m_A a_x + \mu_k m_A g + m_B a_x \Rightarrow T_2 - \mu_k m_A g = (m_A + m_B) a_x$

$\Rightarrow a_x = \dfrac{T_2}{m_A + m_B} - \dfrac{\mu_k m_A}{m_A + m_B} g$

$\Rightarrow a_x = \dfrac{50\,\text{N}}{10\,\text{kg}} - \dfrac{0.50 \times 4.0\,\text{kg}}{10\,\text{kg}} \times 9.8\,\text{m/s}^2$

$a_x = 3.0\,\text{m/s}^2$

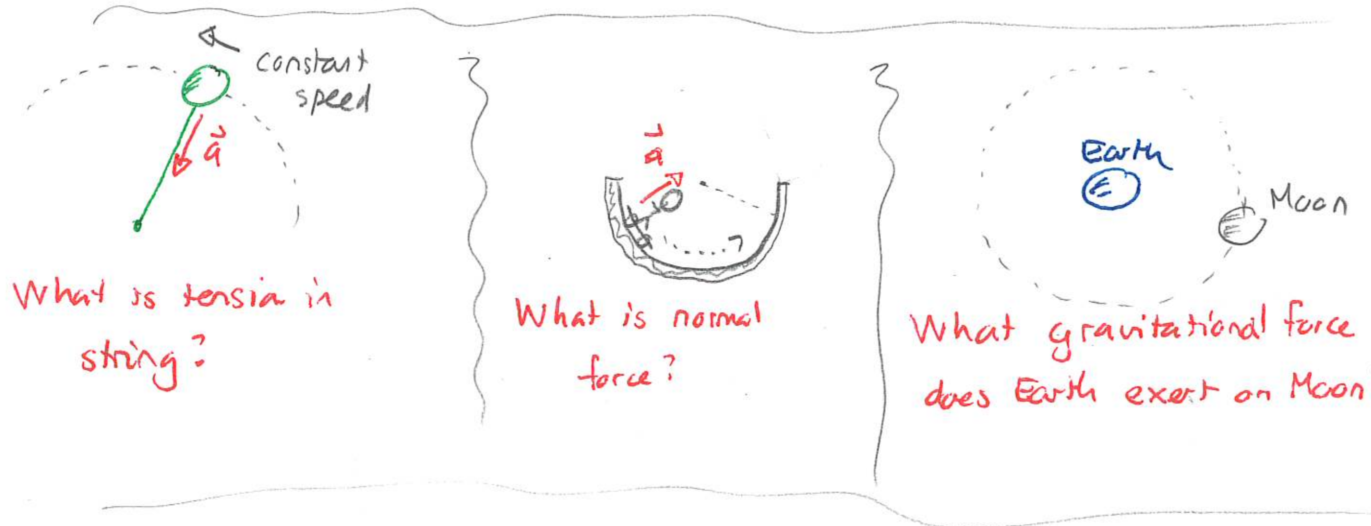
b) $T_1 = \mu_k m_A g + m_A a_x$

$= 0.50 \times 4.0kg \times 9.8m/s^2 + 4.0kg \times 3.0m/s^2$

$= 32N$

Circular Motion

Newton's Laws can be applied to circular motion and ultimately relate the forces on an object in circular motion to its state of motion. Various examples are.



We will mostly focus on uniform circular motion although the analysis will apply to objects on particular points of vertical circular tracks.

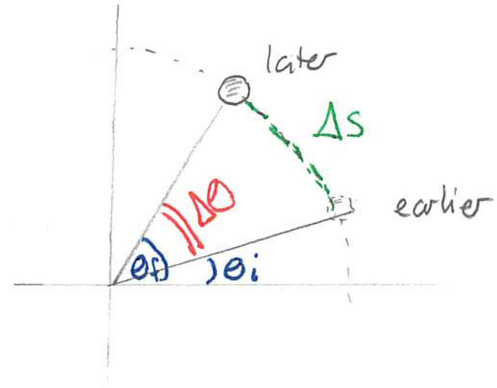
The basic fact from kinematics is:

> An object that undergoes uniform circular motion (constant speed), the acceleration is:
>
> 1) radially inward
> 2) has magnitude $a_c = \frac{v^2}{r}$ where $v =$ speed
>
> $r =$ radius of orbit

We will rephrase this using relevant angular quantities or angular kinematic variables

Angular velocity

We can describe circular motion using



* a co-ordinate system with origin at the circle center
* an angular position, θ, measured in radians from the x-axis

Note that $\boxed{2\pi \text{ radians} = 360^\circ}$

Then the angular displacement of an object over some interval is

$$\boxed{\Delta\theta = \theta_f - \theta_i}$$

This is related to the actual distance traveled (arc length) via

$$\boxed{\Delta s = r\Delta\theta}$$

Then we describe the rate of circular motion via

Angular velocity
~ rate of change of angular position

CONCEPT

The angular velocity of an object is

$$\omega := \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

"Omega"

units rad/s

DEFINITION

Quiz $\rightsquigarrow$ 80% $\lessgtr$ 30% ~ 60%

This can be related to the speed of the object by:

For an object moving in a circle, radius r, the speed is

$$v = \omega r$$

where ω is the angular velocity

Proof

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{r\Delta\theta}{\Delta t} = r \lim \frac{\Delta\theta}{\Delta t}$$

$$= r\omega.$$ ☒

Quiz 3 80% → 90% ≥ 80%

Quiz 4

The acceleration is

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r} \Rightarrow \quad a_c = r\omega^2$$