Fri: HW Ex: 104, 117, 119, 121, 122, 123, 128, 130
5pm

Mon: Warm Up 5 (D2L)

Thurs: Seminar 12:30pm WS 203

Fri: SPS Meeting 12noon Wubben 218

Circular motion

Objects often move in circles and we need to assess kinematics (velocity, acceleration, ...) for such circular motion. Examples include:

1) object swinging on the end of a string
2) orbital motion of celestial objects.

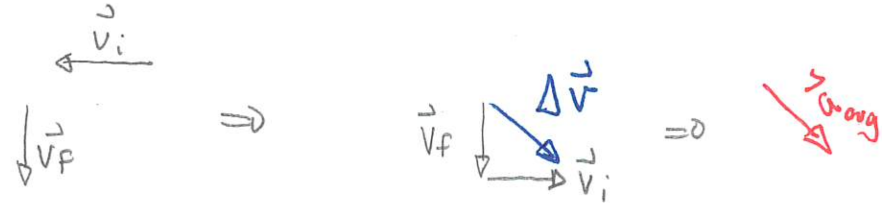
DEMO: The Sky 3D - Solar System Simulator

The tools for analyzing circular motion rest on the same general definitions for two-dimensional motion but they provide specific results.

Quiz 1 70% - 90% & 80%

We analyze this using

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$ AND



This suggests that

$$\boxed{\text{For an object moving in a circle with constant speed, the acceleration points radially inward.}}$$

Uniform circular motion

An important special case of circular motion is uniform circular motion. Here:

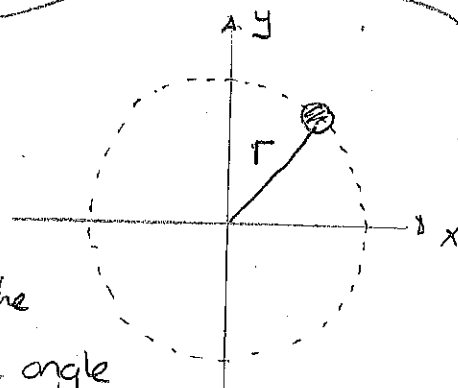
1) the object moves in a circle
2) the speed of the object is constant

The acceleration can be analyzed using

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath}$$

$x = R\cos(\omega t)$

$y = R\sin(\omega t)$



where ω is the rate at which angle is covered in rad/s and $\omega = v/r$

These produce:

For uniform circular motion the acceleration (centripetal acceleration):

1) points radially inward
2) has magnitude

$$a_c = \frac{v^2}{r}$$

where r = radius of orbit

v = speed

125 Merry-go-round

A merry-go-round is a large flat disk that spins around a vertical axis through its center. A child is at the edge of a merry-go-round with radius 3.0 m. The merry-go-round spins so that the child's acceleration is $1.5g$. Determine the period and frequency of orbit for this to occur. *(131F2024)*

Answer: * Get speed from acceleration via $a = v^2/r$

* Period = time for one rotation, T

$\Rightarrow$ covers distance $2\pi r$ in time T

* Get speed $v = \frac{\text{distance}}{\text{period}} = \frac{2\pi r}{T}$.

* Frequency $= f = \frac{1}{T}$.

So $a = \frac{v^2}{r} \Rightarrow v^2 = ar \Rightarrow v = \sqrt{ar}$

$$\Rightarrow v = \sqrt{1.5 g r} = \sqrt{1.5 \times 9.8\,m/s^2 \times 3.0\,m} = 6.6\,m/s$$

Then $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi \times 3.0\,m}{6.6\,m/s}$

$$\Rightarrow T = 2.8\,s$$

Quiz 2 10% – mix

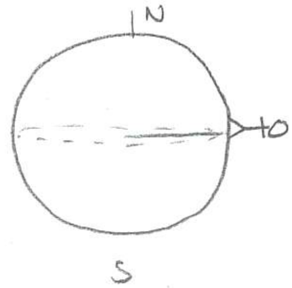
132 Acceleration on Earth's surface

People stand on Earth's surface and are at rest *relative to Earth.* Earth has a radius of 6.4×10^3 km and spins about its poles at a rate of one revolution every 24 hrs. *(131F2024)*

a) Determine the acceleration of a person at Earth's equator.

b) Another person stands at a location much closer to the North pole. Is this person's acceleration the same as, larger than or smaller than that of a person at the equator? Explain your answer.

a)



The person orbits in a circle with radius $R =$ radius of Earth.

The time taken to complete one orbit is

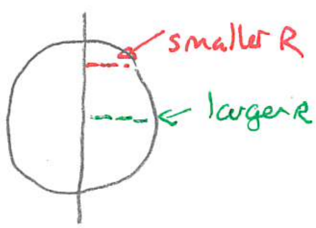
$T = 24\,\text{hrs} \times 3600\,\text{s/hr} = 86400\,\text{s}.$

$$a = \frac{v^2}{R} \quad \text{and} \quad v = \frac{2\pi R}{T}$$

$$\Rightarrow a = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R^2}{T^2 R} \quad \Rightarrow a = \frac{4\pi^2 R}{T^2}$$

$$\text{Thus } a = \frac{4\pi^2 \times 6.4\times 10^6\,\text{m}}{(86400\,\text{s})^2} = 0.034\,\text{m/s}^2$$

b)



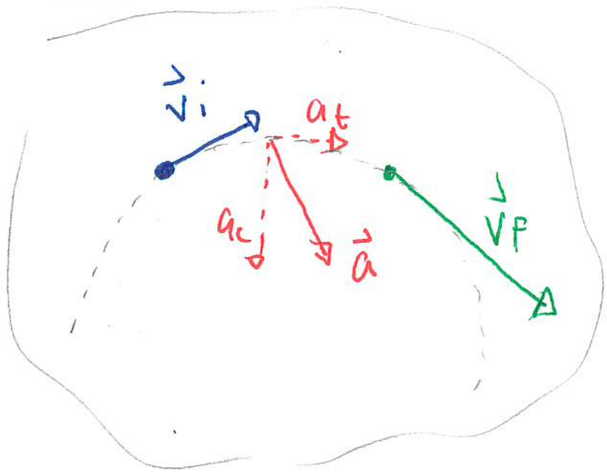
At higher latitudes R is smaller

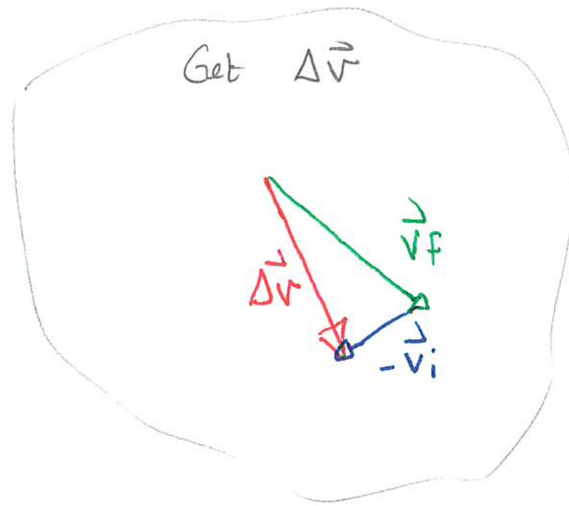
$\Rightarrow a$ is smaller

Non-uniform circular motion

An object moving in a circle can speed up or slow down. The usual methods will yield acceleration

Quiz 3





gives acceleration

Then $\vec{a}$ has two components:

a_c = centripetal accel (radially inward) → describes circular motion

a_t = tangential accel (tangent to circle) → describes speed change