Fri: HW by 5pm

Mon: Warm Up 4 (D2L) Group Exercise

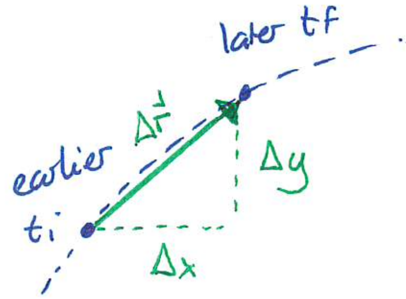
Tues: Discussion /quiz

EX:

## Velocity in Two Dimensions

Position in two dimensions is described by a vector and it follows that:

* displacement is a vector
* velocity is a vector




Specifically

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\jmath}$$

$$= \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

Thus

$$\boxed{\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}}$$

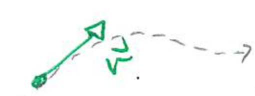
and this has properties:

1) magnitude of $\vec{v}$ is speed

$$v = \sqrt{v_x^2 + v_y^2}$$

speed

2) direction of $\vec{v}$ is tangent to the trajectory in the direction of motion




DEMO: PhET Ladybug 2D → show $\vec{v}$ vector

→ Ellipse ...

Quiz 1 40% – 80% // 60% – 80%

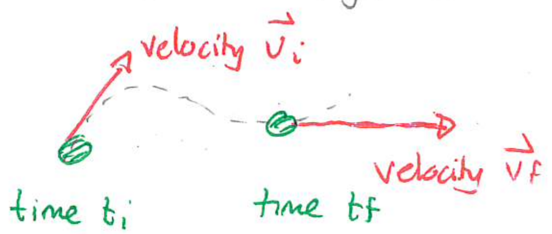
Quiz 2 30% – 80% // 50% – 80%

Quiz 3 90% // 70% – 90%

## Acceleration in two dimensions

Acceleration describes the rate of change of velocity. Again a preliminary definition is average acceleration

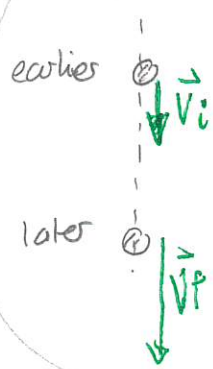
Observe an object at two instants ⇝ the average acceleration between the two instants is



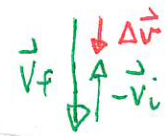

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

**Example:** A ball drops toward Earth. Determine the direction of the acceleration vector as it falls

**Answer:** ① Sketch trajectory and velocity vectors at two instants




→ ② Find $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$




→ ③ Direction of $\vec{a}_{avg}$ is same as direction of $\Delta\vec{v}$ (since $\Delta t > 0$)

$\downarrow \vec{a}_{avg}$

Quiz 4 60% – 90% // 60% – 70%

This eventually leads to instantaneous acceleration

The acceleration of an object is $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

## Properties of acceleration in two dimensions

1) acceleration is a vector.

$$\boxed{\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath} \quad \text{where} \quad a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}}$$

2) the direction of acceleration is not always the same as the direction of motion. It does correspond to the direction of $\Delta \vec{v}$. Acceleration describes how velocity changes.

## Constant acceleration

An important category of motion is that where acceleration is constant. Then over any time interval it's exactly true that

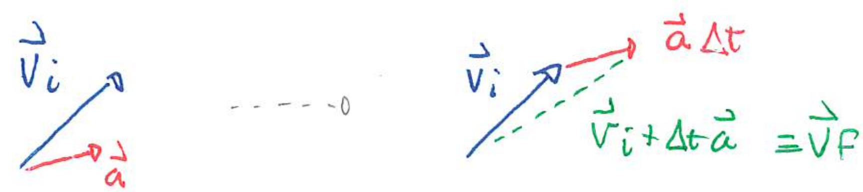
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = \vec{a}\, \Delta t$$

$$\Rightarrow \vec{v}_f - \vec{v}_i = \vec{a}\, \Delta t.$$

Thus, for constant acceleration

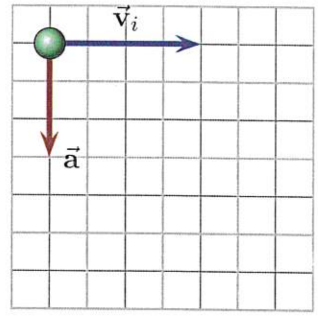
$$\vec{v}_f = \vec{v}_i + \vec{a}\, \Delta t$$

We can use this to assess the trajectory of an object or how its state of motion changes. We have to add vectors



## 100 Constant vertical acceleration

A ball launches off a horizontal surface. At the moment of launch its velocity is $\vec{\mathbf{v}}_i$. At all later times it accelerates with a constant acceleration, $\vec{\mathbf{a}}$. The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). *(131F2024)*

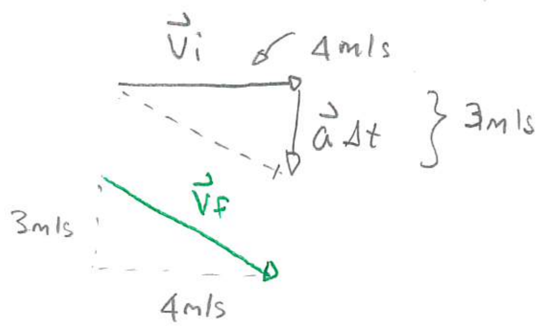



a) Draw, as accurately as possible, the velocity vector, $\vec{\mathbf{v}}_f$, at an instant 1.0 s after the initial instant.

b) Using $\vec{\mathbf{v}}_f$ describe whether the object is moving faster at the 1.0 s instant than at the initial instant.

c) Using $\vec{\mathbf{v}}_f$ describe the direction in which the object is moving at the 1.0 s instant.

Answer: a) $\vec{V}_f = \vec{V}_i + \vec{a}\underbrace{\Delta t}_{1s}$

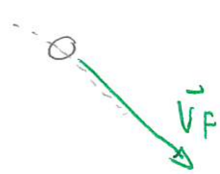



b) $V_f = \sqrt{(3m/s)^2 + (4m/s)^2} = 5m/s$

$V_i = 4m/s$ so $V_f > V_i$

moves faster.

c) velocity vector is tangent to trajectory $\Rightarrow$ moves right/down



## Kinematic equations

When the acceleration is constant

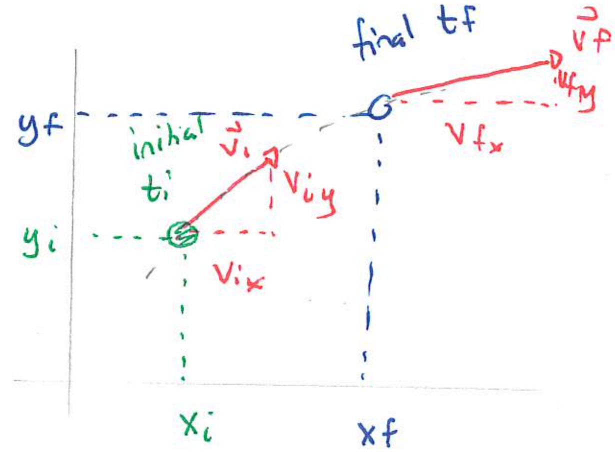
$$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t \quad \Rightarrow \quad v_{fx} = v_{ix} + a_x \Delta t$$
$$v_{fy} = v_{iy} + a_y \Delta t$$

shows that we can analyze the horizontal + vertical components independently. The usual integration steps give:

If acceleration is constant then

| Only horizontal | Only vertical |
|---|---|
| $v_{fx} = v_{ix} + a_x \Delta t$ | $v_{fy} = v_{iy} + a_y \Delta t$ |
| $x_f = x_i + v_{ix}\Delta t + \frac{1}{2} a_x \Delta t^2$ | $y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a_y \Delta t^2$ |
| $v_{fx}^2 = v_{ix}^2 + 2a_x (x_f - x_i)$ | $v_{fy}^2 = v_{iy}^2 + 2a_y (y_f - y_i)$ |

only time in common




## Projectile motion

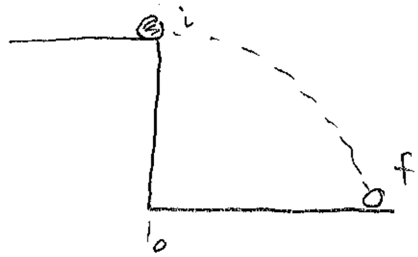
Projectile motion is that where only Earth's gravity acts on an object.
Experimental observations show that:

> The acceleration of a projectile is constant with components
>
> $a_x = 0\,m/s^2$
>
> $a_y = -g = -9.80\,m/s^2$

## 108 Running off a roof

A person runs with speed 8.0 m/s off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land. *(131F2024)*

a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground.




List as many of the variables as possible. Use the format:

| | | | |
|---|---|---|---|
| $t_i =$ | 0 s | $t_f =$ | |
| $x_i =$ | 0 m | $x_f =$ | |
| $y_i =$ | 3.0 m | $y_f =$ | 0 m |
| $v_{ix} =$ | 8.0 m/s | $v_{fx} =$ | |
| $v_{iy} =$ | 0 m/s | $v_{fy} =$ | |
| $a_x =$ | 0 m/s² | $a_y =$ | −9.80 m/s² |

b) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{v}_i$. Enter these in the list above.

$\vec{V}_i \longrightarrow$

$V_{ix} = 8.0\,m/s$

$V_{iy} = 0\,m/s$

c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it.

Need Xf:

$$X_f = \cancel{X_i}^{0} + V_{ix}\Delta t + \frac{1}{2}\cancel{a_x}^{0}\Delta t^2$$

$$X_f = V_{ix}\Delta t$$

You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?

need Δt

d) Use the vertical aspects of the object's motion to solve for this other unknown variable and use this result to answer the question of this problem.

$$y_f = y_i + \overset{0\,m/s}{\cancel{v_{iy}}}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0\,m = 3.0\,m + \frac{1}{2}(-9.8\,m/s^2)(\Delta t)^2$$

$$\Rightarrow -3.0\,m = -4.9\,m/s^2\,(\Delta t)^2$$

$$\Rightarrow (\Delta t)^2 = \frac{3.0\,m}{4.9\,m/s^2}$$

$$= 0.61\,s^2 \Rightarrow \Delta t = \sqrt{0.61\,s} = 0.78\,s$$

$$\Rightarrow X_f = 8.0\,m/s \times 0.78\,s = 6.3\,m$$

Suppose that the person ran and jumped from the building at an angle of 30° above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?

e) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{v}_i$. Reconstruct the list of variables for the problem.

f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.

g) Use the kinematic equations to determine the maximum height that the person reaches.