Phys 131 2024

Lecture 8

Fri: HW by 5pm

Ex 76, 77, 78, 87, 88a, 89, 90, 91

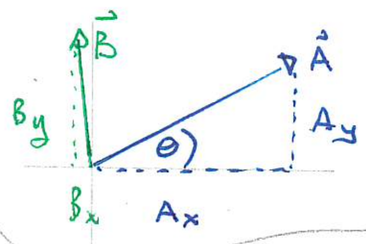
Mon: Warm Up 4 (D2L)

Thurs: Seminar WS 203 12:30 - 1:30

Vector algebra using components

Vectors can be added using components.

Task: Given vectors $\vec{A}, \vec{B}$ determine $\vec{D} = \alpha\vec{A} + \beta\vec{B}$



→ **Strategy:** Get components of $\vec{A}, \vec{B}$ and use these to find components of $\vec{D}$

Components of vectors $\vec{A}, \vec{B}$

$A_x = A\cos\theta$ $\qquad B_x =$

$A_y = A\sin\theta$ $\qquad B_y = \ldots$

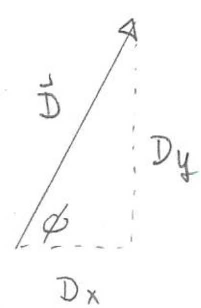
Use trig to get components. Insert ± signs

Compute components of $\vec{D}$:

$D_x = \alpha A_x + \beta B_x$

$D_y = \alpha A_y + \beta B_y$

→ Reconstruct $\vec{D}$ from its components



magnitude

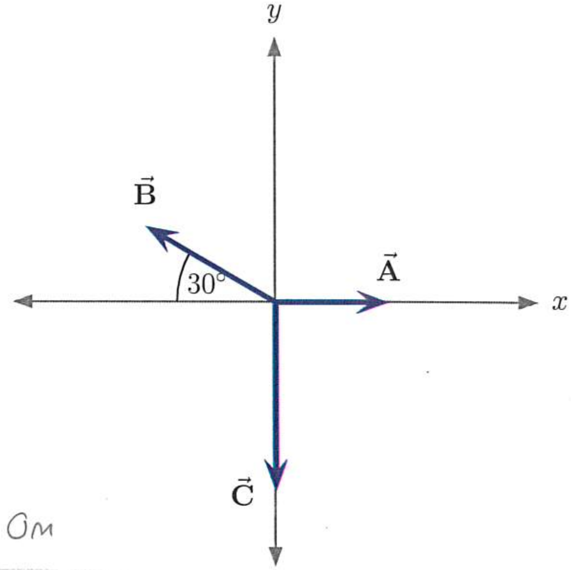
$$D = \sqrt{D_x^2 + D_y^2}$$

direction

$$\phi = \arctan\frac{D_y}{D_x}$$

80 Vector components: algebraic, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Their magnitudes are $A = 15\,\text{m}, B = 20\,\text{m}$ and $C = 25\,\text{m}$. *(131F2024)*



a) Determine the x and y components of each vector.

b) Determine the components of $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. Determine the magnitude of $\vec{\mathbf{D}}$.

Answer: a)

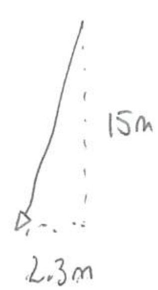
$A_x = 15\,\text{m}$	$A_y = 0\,\text{m}$
$B_x = -B\cos 30^\circ$	$B_y = B\sin 30^\circ$
$= -20\,\text{m}\cos 30^\circ$	$= 20\,\text{m}\sin 30^\circ$
$B_x = -17.3\,\text{m}$	$B_y = 10\,\text{m}$
$C_x = 0\,\text{m}$	$C_y = -25\,\text{m}$

b) $D_x = A_x + B_x + C_x$

$= 15\,\text{m} - 17.3\,\text{m} + 0\,\text{m} = -2.3\,\text{m}$

$D_y = A_y + B_y + C_y$

$= 0\,\text{m} + 10\,\text{m} - 25\,\text{m} = -15\,\text{m}$



$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-2.3\,\text{m})^2 + (-15\,\text{m})^2}$$

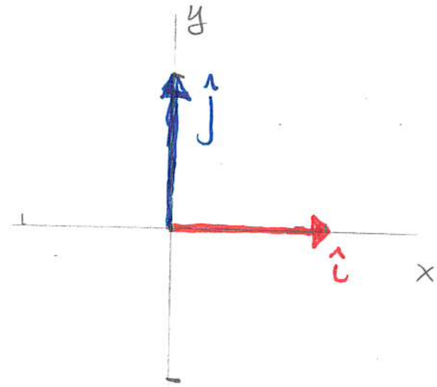
$$= 15.2\,\text{m}$$

Unit Vectors

A vector can be specified by its components. We would like to do this in ways that allow for algebraic manipulation. One possibility involves using special unit vectors. These are:

$\hat{\imath}$ = unit vector along x-axis

$\hat{\jmath}$ = unit vector along y-axis



We can construct any vector from multiples of these unit vectors.

Let $\vec{A}$ be a vector in two dimensions. Then

$$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath}$$

where A_x = x-component of $\vec{A}$
A_y = y-component of A

numbers that could be positive or negative.

Slide 1

Slide 2

Slide 3

In the previous example,

$$\vec{A} = 15\text{m}\,\hat{\imath}$$

$$\vec{B} = -17.3\text{m}\,\hat{\imath} + 10\text{m}\,\hat{\jmath}$$

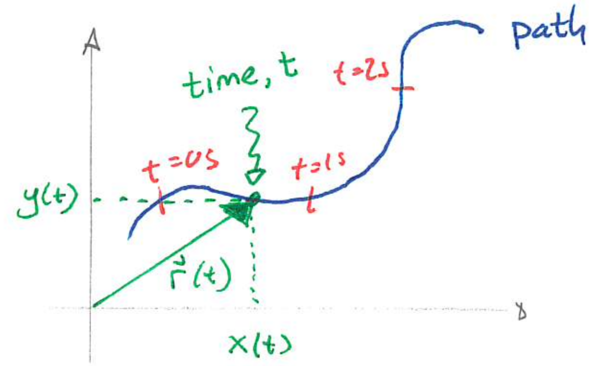
$$\vec{C} = -25\text{m}\,\hat{\jmath}$$

$$\vec{A}+\vec{B}+\vec{C} = 15\text{m}\,\hat{\imath} - 17.3\text{m}\,\hat{\imath} + 10\text{m}\,\hat{\jmath} - 25\text{m}\,\hat{\jmath}$$

$$= -2.3\text{m}\,\hat{\imath} - 15\text{m}\,\hat{\jmath}$$

Kinematics in Two Dimensions

We now use vectors to extend the language of one-dimensional kinematics to two dimensions. Graphically we can represent the motion via a trajectory or path in two dimensional space



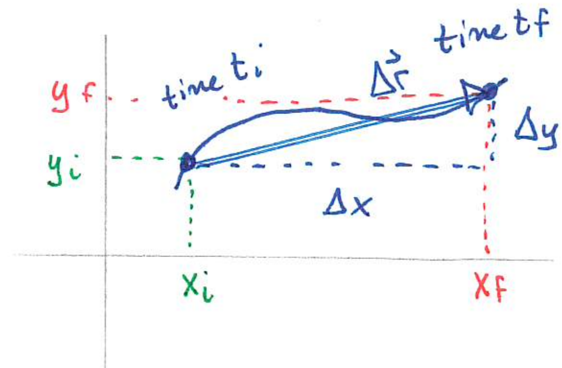
The time can be indicated at various path locations. We can then track the location mathematically via:

$x(t)$ = horizontal position function
$y(t)$ = vertical position function

→ position vector

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

We use these to defined a velocity vector. We start with an average velocity vector that has two components. The procedure is:



Observe object at two instants.

	earlier	later
time	t_i	t_f
horiz pos	x_i	x_f
vert pos	y_i	y_f

↳ Displacement vector

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$\Delta x = x_f - x_i$$

$$\Delta y = y_f - y_i$$

→ Average velocity vector from t_i to t_f is

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\,\hat{i} + \frac{\Delta y}{\Delta t}\,\hat{j} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous velocity is a vector

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\,\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\,\hat{j}$$

$$\vec{v} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j}$$

Quiz 1   30% $\rightsquigarrow$ 50% $\gtrless$ 30%

We can see that

> Velocity is a vector
>
> $$\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}$$
>
> where the velocity components are.
>
> $$v_x = \frac{dx}{dt} \quad \text{and} \quad v_y = \frac{dy}{dt}.$$

Then, for the velocity vector:

> The velocity vector has:
>
> 1) magnitude $v = \sqrt{v_x^2 + v_y^2} \equiv$ speed
> 2) direction: tangent to trajectory along direction of motion
>
> 
>

Quiz 2

Quiz 3

Quiz 4