Phys 131 2024

Lecture 3

Monday: HW due by 5pm

131 Ex: 25, 26, 27, 29, 33, 34, 35, 37

* Return on paper
* Complete solutions to each problem - explain your answer!
* do NOT use Internet resources
* counts ~~12p~~ 14pts

Group exercise

no grade for this...

Tuesday Will be discussion /quiz

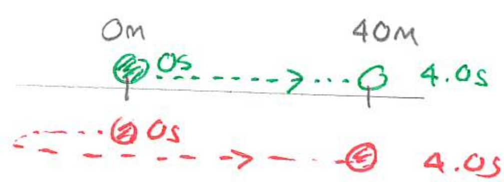
* Fewer problems than usual (to catch up...)

SPS mtg

Weds 28 Aug.

Instantaneous velocity

Average velocity quantifies the rate at which position changes via $V_{avg} = \Delta x / \Delta t$. But this is imperfect since it cannot distinguish between situations such as that illustrated.



We can refine this by considering portions of the motion over very small intervals, eventually arriving at the concept of velocity.

$V_{avg} = \frac{40m}{4.0s} = 10m/s$ both

DEMO: Moving Man PHET → Charts

→ $x_0 = +10$, $V_0 = -6$, $a = 2$ } Show position vs. time

What would the velocity at 4.0s be? We can consider intervals of

4.0s → 5.0s

4.0s → 4.5s

etc...

the data would be

t_i	t_f	x_i	x_f	Δt	Δx	V_{avg}
4.00s	5.00s	2.0m	5.0m	1.00s	3.0m	3.0m/s
4.00s	4.50s	2.0m	3.25m	0.50s	1.25m	2.5m/s

Slide of data

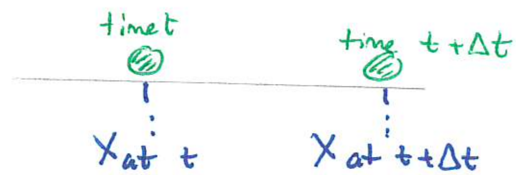
We see that the average velocity approaches a fixed value (2.0m/s). This will be the instantaneous velocity at 4.00s.

These lead to:

(Instantaneous) velocity ⟶ rate at which position changes at one instant

The definition is

The instantaneous velocity of an object at time t is the limiting value of the average velocity over the interval $t \to t+\Delta t$ as $\Delta t \to 0$. Thus



$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \text{limit} \frac{x_{\text{at } t+\Delta t} - x_{\text{at } t}}{\Delta t}$$

units: m/s

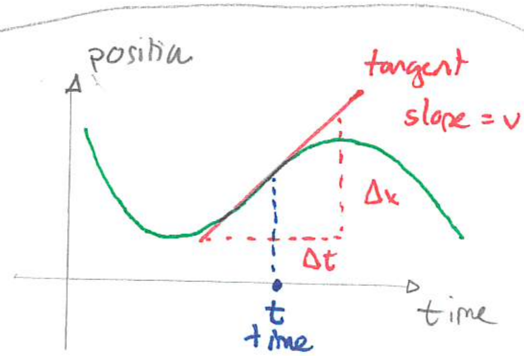
How can one calculate this?

1) approximately via a table of position versus time data
2) exactly using calculus if we know x as a function of time.

Additionally we can show:

Given a graph of position versus time the velocity a time t is

V = slope of tangent to x vs t graph at time t



Warm Up 1

Then the instantaneous velocity allows a definition of speed

| Instantaneous speed = s = magnitude of velocity |

Quiz! 90% ≷ 90%

Note that velocity has two aspects:

magnitude of velocity = speed

sign of velocity

Position from velocity

We often need to infer position information from velocity information. For

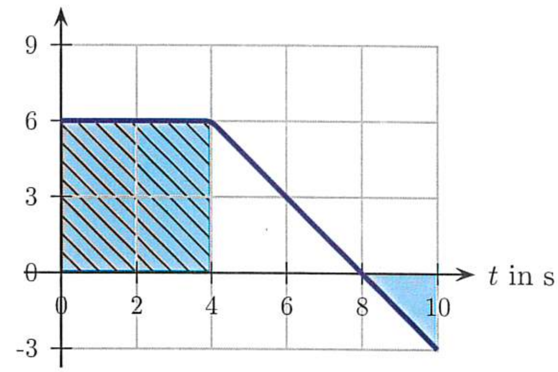
!! UNIFORM MOTION ONLY !! — given velocity v ⇒ change in position during time interval Δt is $\Delta x = v\Delta t$

This lets us determine displacement from velocity and the duration of the interval. But what if the velocity is not constant

VELOCITY NOT CONSTANT ⇒ $\Delta x \neq v\Delta t$

The mathematical procedure for determining velocities involves areas.

31 Crawling slug



A slug crawls along a straight wire, starting at $x = 0.0\,\text{m}$ at $t = 0.0\,\text{s}$. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. *(131F2023)*

a) Determine the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$.

b) How is the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$ related to the shaded area *between the graph and the horizontal axis* ($v = 0.0\,m/s$)?

c) Assuming that the answer to the previous question is true in general, determine the displacement of the slug from $t = 4.0\,\text{s}$ to $t = 8.0\,\text{s}$.

d) Is the displacement of the slug from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$ positive or negative? How might this relate to the shaded area from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$?

Ans: a) Here v is constant. So

$\Delta x = v\Delta t = 6.0\,m/s \times 4s = 24m$

b) area $= 6m/s \times 4m = 24m$. They are the same

c) total area = square area + triangle area ⟶ $\frac{1}{2}bh$

$= 24m + \frac{1}{2} \times 4s \times 6m/s = 24m + 12m = 36m$

d) negative because $v < 0$ ⟹ moves left

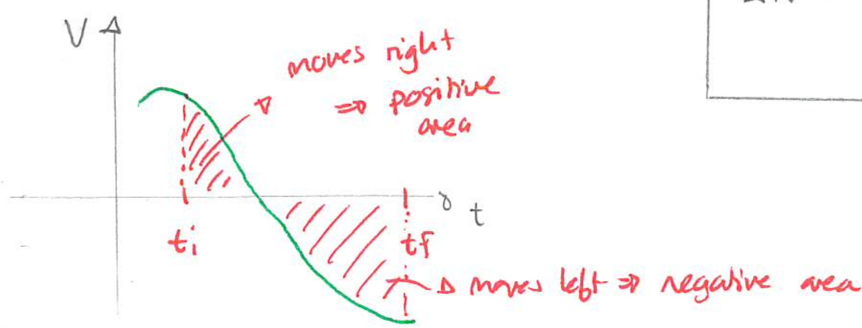
area $= \frac{1}{2}bh = \frac{1}{2}\, 2s \times 3m/s = 3m$

so displacement $= -3m$

In general

Given a graph of velocity versus time ⇝ Displacement from t_i to t_f is

$\Delta x =$ area between graph and t-axis from t_i to t_f



Quiz 2

Calculating velocity from position

Calculus provides the following:

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$ "derivative of x with respect to t"

and also various rules for calculating derivatives. For example

If $x = at^n$ where a, n are constants then $\frac{dx}{dt} = n a t^{n-1}$

36 Velocity as a derivative, 1

Suppose that the position of an object is

$$x = (5\,\mathrm{m/s^2})\ t^2 + (3\,\mathrm{m/s})\ t$$

Determine the velocity of the object at $t = 3\,\mathrm{s}$. *(131Sp2023)*

Answer: $v =$ deriv of x w.r.t t

$= $ deriv of $5t^2$ + deriv $3t^1$

$= 5 \times 2t^{2-1} + 3 \cdot 1\, t^{1-1}$

$= 10t + 3\,\mathrm{m/s}$

At 3s $\quad v = 10\,\mathrm{m/s^2} \times 3\,\mathrm{s} + 3\,\mathrm{m/s}$

$= 33\,\mathrm{m/s}$