Fundamental Mechanics: Class Exam 2

18 October 2024

Name: SOLUTION **Total:** /70

Instructions

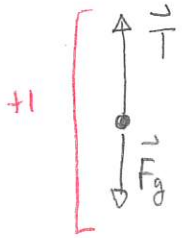
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80\,\mathrm{m/s^2}$$

Question 1

A 8.0 kg fish hangs at rest from a rope. Starting at some initial moment a person pulls vertically upward on the rope with force 95 N. The person pulls for 0.50 s. Determine how far the fish moves while the person pulls it.



+1

$\Sigma F_{iy} = m a_y$] +1

$T - mg = ma_y$

$\Rightarrow 95\,N - 8 \times 9.8\,m/s^2 = 8\,kg\; a_y \Rightarrow 16.6\,N = 8\,kg\; a_y$

$\Rightarrow a_y = 2.1\,m/s^2$] +3

$y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a_y \Delta t^2$] +1

$y_f - y_i = \frac{1}{2} a_y \Delta t^2$

$\Delta y = \frac{1}{2}\, 2.1\,m/s^2 \times (0.50\,s)^2 = 0.26\,m$] +2

$v_{iy} = 0$

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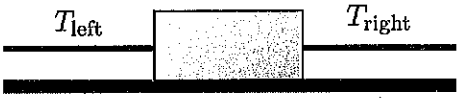
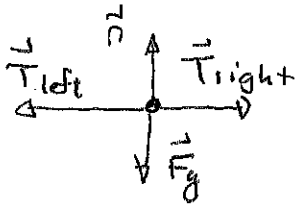
Question 2

Two ropes, pulled by people, are attached to a box that slides across a frictionless surface. After a while, the box moves right with decreasing speed. While this occurs, which of the following (choose one) is true regarding the magnitudes of the tensions in the ropes?



i) The tension in the right rope must equal that in the left.
ii) The tension in the right rope must be larger than in the left.
iii) The tension in the left rope must be larger than in the right.

Briefly explain your answer.



Acceleration is left since $\vec{v}_i$ → , $\vec{v}_f$ →

$\Rightarrow \Delta\vec{v}$ ←

$$\Sigma F_{ix} = ma_x$$

$$T_{right} - T_{left} = ma_x$$

$$T_{right} = T_{left} + ma_x$$ (a_x negative)

$$\Rightarrow T_{right} < T_{left}$$

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Question 3

A 0.010 kg coin is at rest on a table. Which of the following (choose one) is true?

i) The coin does not exert a force on Earth.
ii) The coin exerts a force of 9.8 N on Earth.
iii) The coin exerts a force of 0.098 N on Earth.
iv) The coin exerts a force of 0.010 N on Earth.

Earth exerts a gravitational force $F_g = mg = 0.010\text{kg} \times 9.8\text{M/s}^2 = 0.098\text{N}$

By Newton's third law coin exerts same force on Earth.

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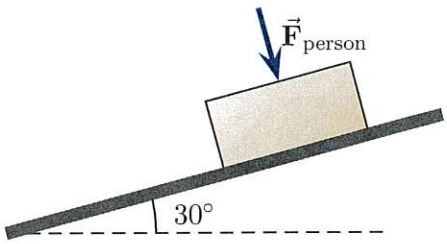
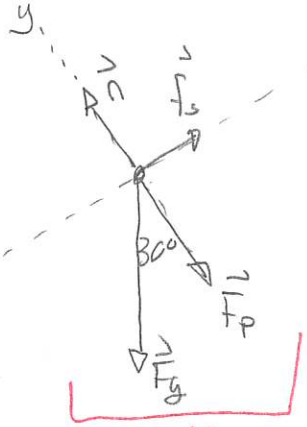
Question 4

A 50 kg box is on an inclined ramp. The coefficient of kinetic friction between the block and ramp is 0.30 and the coefficient of static friction is 0.40. A person pushes perpendicular to the surface and the block stays at rest; without this force the block would slip down the ramp. Determine the minimum force that the person must exert to keep the block at rest.





+1

$\sum F_{ix} = ma_x = 0$

$\sum F_{iy} = ma_y = 0$] +1

$F_g = mg$

$f_s \leq \mu_s n$] +1

	x	y
$\vec{F}_g$	$-mg\sin 30^\circ$	$-mg\cos 30^\circ$
$\vec{n}$	0	n
$\vec{F}_P$	0	$-F_P$
$\vec{f}_s$	$+f_s$	0

+3

$\sum F_{ix} = 0$

$\Rightarrow +f_s - mg\sin 30^\circ = 0$

$\Rightarrow f_s = mg\sin 30^\circ$] +2

correct direction +1

$\sum F_{iy} = 0 \Rightarrow n - F_P - mg\cos 30^\circ = 0 \Rightarrow n = F_P + mg\cos 30^\circ$

Now $f_s \leq \mu_s n \Rightarrow mg\sin 30^\circ \leq \mu_s [F_P + mg\cos 30^\circ]$

$\Rightarrow mg\sin 30^\circ - \mu_s mg\cos 30^\circ \leq \mu_s F_P$

$\Rightarrow \frac{mg}{\mu_s}[\sin 30^\circ - \mu_s \cos 30^\circ] \leq F_P$

+3

$\Rightarrow F_P \geq \frac{50\text{kg} \times 9.8\text{m/s}^2}{0.40}[\sin 30^\circ - 0.40\cos 30^\circ]$

$\Rightarrow F_P \geq 188\text{ N}$

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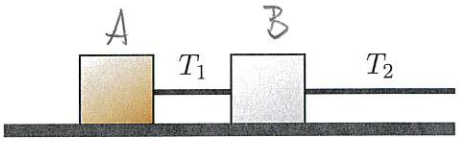
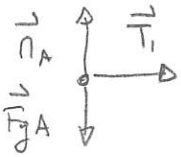
Question 5

Two identical blocks, each with mass 20 kg, are connected by a rope and are pulled to the right across a frictionless surface. The acceleration of both blocks is observed to be $0.40\,\mathrm{m/s^2}$. Determine the tension T_1 **and** describe whether T_2 is larger or smaller than T_1.



On block A



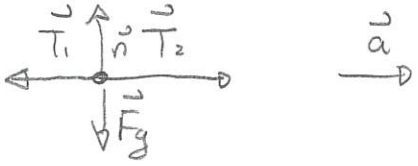
$\sum F_{ix} = m_A a$

$\Rightarrow T_1 = m_A a$

$= 20\,\mathrm{kg} \times 0.40\,\mathrm{m/s^2}$

$\Rightarrow T_1 = 8.0\,\mathrm{N}$

On blck B



$\sum F_{ix} = m_B a$

$\Rightarrow T_2 - T_1 = m_B a \geq 0$

$\Rightarrow T_2 > T_1$

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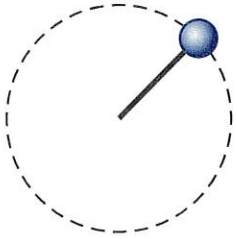
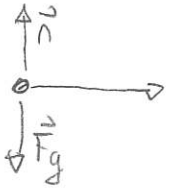
Question 6

A ball, at the end of a string, initially swings with a constant speed of 2.0 m/s in a circle on a frictionless horizontal surface. Suppose that you wanted the speed of the ball to increase to 6.0 m/s without changing the length of the string. Explain as precisely as possible how you would have to adjust the tension (e.g. keep it the same, larger, smaller, ...) so that you could get its speed to increase as described.





$\vec{a} = 0$

$\sum F_{ix} = ma_x$ $\quad a = v^2/r$

$\Rightarrow T = mv^2/r$

The speed goes from $v_0 \to 3v_0$. Thus

$T \to \frac{m}{r} v_0^2 \to \frac{m}{r}(3v_0)^2 = 9\,\frac{mv_0^2}{r}$

$\Rightarrow$ Tension increases by a factor of 9

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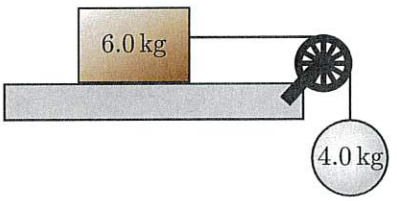
Question 7

A 6.0 kg wooden block lies on a horizontal rough surface. The coefficient of kinetic friction between the block and the surface is 0.45. The block is connected by a massless string, that runs horizontally over a massless pulley, to a suspended 4.0 kg ball. The block moves left.

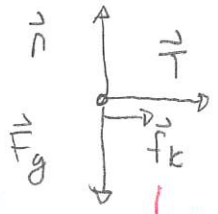


a) Determine the acceleration of the block.

at least once (+1)

Block (m_1) $\Sigma F_{ix} = m_1 a_{1x} \Rightarrow T + f_k = m_1 a_{1x} \Rightarrow T + \mu_k n = m_1 a_{1x}$ (+1)

(+1)



$\Sigma F_{iy} = m_1 a_{iy} = 0 \Rightarrow n - m_1 g = 0 \Rightarrow n = m_1 g$ (+1)

combining these gives $T + \mu_k m_1 g = m_1 a_{1x}$ (+2)

↳ friction direction incorrect (-2)

Ball (m_2) $\Sigma F_{iy} = m_2 a_{2y} \Rightarrow T - m_2 g = m_2 a_{2y}$ (+2)

(+1)

Let a be the magnitude of the acceleration $T + \mu_k m_1 g = m_1 a$

$T - m_2 g = -m_2 a$ → negative accel (+1)

(+3)

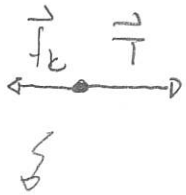
$\Rightarrow T = m_2 g - m_2 a$

Thus $m_2 g - m_2 a + \mu_k m_1 g = m_1 a \Rightarrow (m_2 g + \mu_k m_1 g) = (m_1 + m_2) a$

$\Rightarrow a = \frac{(m_2 + \mu_k m_1)}{m_1 + m_2} g = \frac{4.0\,\text{kg} + 0.45 \times 6\,\text{kg}}{10\,\text{kg}} 9.8\,\text{m/s}^2$

$a = 6.6\,\text{m/s}^2$

Question 7 continued ...



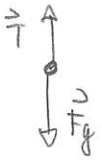
Now $\vec{f}_k$ and $\vec{T}$ are opposite so the net force on the block is smaller $\Rightarrow$ a smaller

b) The block eventually moves right. Which of the following (choose one) is true about the magnitude of the acceleration, a?

(+3)

i) When moving right a is the same when moving left.
ii) When moving right a is smaller than when moving left.
iii) When moving right a is larger than when moving left.

c) The block eventually moves right. Which of the following (choose one) is true about the tension in the string, T?



$m_2g - T = m_2a$

(+2)

i) When moving right T is the same when moving left.
ii) When moving right T is smaller than when moving left.
iii) When moving right T is larger than when moving left.

$T = m_2g - m_2a$

Acceleration is reduced $\Rightarrow$ T larger.

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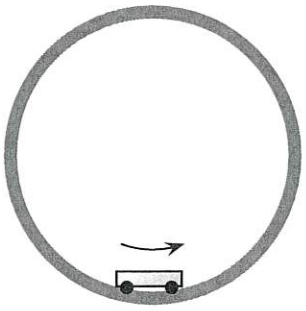
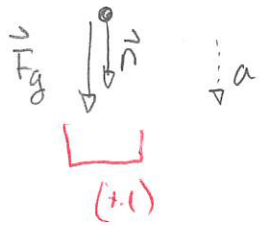
Question 8

A cart of mass m moves around a vertical loop with radius r that is just above Earth's surface. The cart maintains a constant speed v.



a) Determine an expression (with no symbols other than m, g, v, r and constants) for the normal force on the cart when it is at the top of the loop.



(+1)

$\Sigma F_y = ma$ (+1)

$\Rightarrow n + mg = mv^2/r$ (+2) (+1)

$n = m\left(\frac{v^2}{r} - g\right)$ (+1)

b) Determine an expression for the minimum speed such that the cart can do this without leaving the loop at the top.

(+2)

At the minimum speed $n \to 0$ $\Rightarrow \frac{v^2}{r} = g$

$\Rightarrow v = \sqrt{gr}$ (+2)

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