

Fundamental Mechanics: Class Exam 1

20 September 2024

Name: SOLUTION

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Instructions

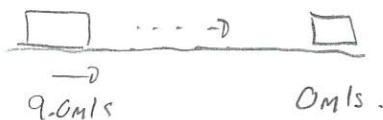
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80 \text{ m/s}^2$$

Question 1

A cart slides on a horizontal surface. At an initial instant it moves with speed 9.0 m/s. Immediately after this instant it slows (with constant acceleration) to a stop in 1.5 s. Determine the distance traveled by the cart from the initial instant until it stops.



$$\begin{aligned} t_i &= 0 & t_f &= 1.5 \text{ s} \\ x_i &= 0 & x_f &= \\ v_i &= 9 \text{ m/s} & v_f &= 0 \\ a &= ? & & \end{aligned}$$

First acceleration

$$\begin{aligned} v_f &= v_i + a \Delta t &] +1 \\ 0 &= 9.0 \text{ m/s} + a (1.5 \text{ s}) \\ a &= \frac{-9.0 \text{ m/s}}{1.5 \text{ s}} = -6.0 \text{ m/s}^2 & } +2 \end{aligned}$$

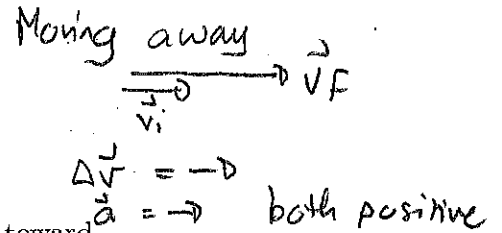
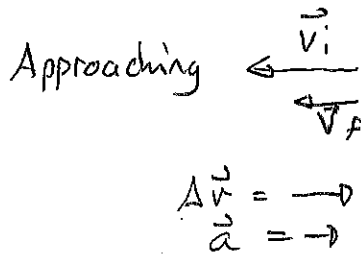
Then displacement

$$\begin{aligned} x_f &= x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2 &] +1 \\ x_f &= 9.0 \text{ m/s} \times 1.5 \text{ s} + \frac{1}{2} (-6.0 \text{ m/s}^2) (1.5 \text{ s})^2 \end{aligned}$$

$$x_f = 13.5 \text{ m} - 6.75 \text{ m} \quad /8$$

$$x_f = 6.75 \text{ m} \quad +3$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$



Question 2

An atom initially travels left (as shown) along a straight line toward point P, slowing as it approaches P. It then reverses direction and travels right, speeding up as it moves away from P. In the following, rightward motion is positive. Which of the following (choose one) is true regarding the acceleration of the atom, a ?

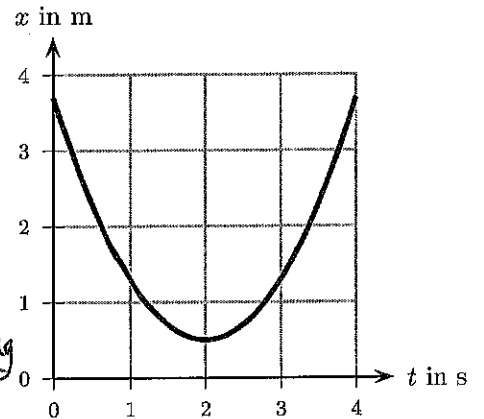


- i) $a > 0$ as it approaches P; $a > 0$ as it moves away from P.
- ii) $a > 0$ as it approaches P; $a < 0$ as it moves away from P.
- iii) $a < 0$ as it approaches P; $a > 0$ as it moves away from P.
- iv) $a < 0$ as it approaches P; $a < 0$ as it moves away from P.

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Question 3

An ant walks along a straight stick. The graph illustrates the ant's position vs time. During the period from 0s to 4s is the acceleration of the ant negative, positive or zero? Explain your answer.



$v = \text{slope } x \text{ vs } t$

before 2s $v < 0$ and constantly increasing

at 2s $v = 0$

after 2s $v > 0$ and constantly increasing


$\Rightarrow \boxed{a > 0}$

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Question 4

A rock is thrown vertically downwards from a window. It leaves the hand at a height of 9.0 m above the ground and hits the ground 0.75 s later.

a) Determine the speed with which the rock was thrown.




$t_i = 0 \quad t_f = 0.75 \text{ s}$
 $y_i = 9.0 \text{ m} \quad y_f = 0 \text{ m}$
 $v_i = ? \quad v_f = ?$

$a = -g = -9.80 \text{ m/s}^2$

$y_f = y_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$
 $0 = 9.0 + v_i \Delta t - \frac{1}{2} g \Delta t^2$
 $\Rightarrow v_i \Delta t = \frac{1}{2} g \Delta t^2 - y_i$
 $\Rightarrow v_i = \frac{1}{2} g \Delta t - \frac{y_i}{\Delta t}$
 $\Rightarrow v_i = \frac{9.8 \text{ m/s}^2 \times 0.75 \text{ s}}{2} - \frac{9.0 \text{ m}}{0.75 \text{ s}}$
 $= 3.675 \text{ m/s} - 12 \text{ m/s}$
 $\Rightarrow v_i = -8.3 \text{ m/s}$

b) Suppose the rock were thrown upwards with the same speed. Determine the time that it would take to hit the ground.



$t_i = 0 \quad t_f = ?$
 $y_i = 9.0 \text{ m} \quad y_f = 0 \text{ m}$
 $v_i = 8.3 \text{ m/s} \quad v_f = ?$

$a_y = -g = -9.8 \text{ m/s}^2$

$y_f = y_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$
 $0 \text{ m} = 9.0 \text{ m} + 8.3 \text{ m/s} \Delta t - \frac{9.8 \text{ m/s}^2}{2} \Delta t^2$
 $\Rightarrow -4.9 \text{ m/s}^2 \Delta t^2 + 8.3 \text{ m/s} \Delta t + 9.0 \text{ m} = 0$
 $\Delta t = \frac{-8.3 \text{ m/s} \pm \sqrt{(8.3 \text{ m/s})^2 - 4(9.0 \text{ m})(-4.9 \text{ m/s}^2)}}{2(-4.9 \text{ m/s}^2)}$

$$\Delta t = \frac{-8.3 \text{ m/s} \pm 15.675 \text{ m/s}}{-9.8 \text{ m/s}^2}$$

$$\Delta t = -0.75 \text{ s} \quad \text{or} \quad 2.48 \text{ s}$$

can only be this

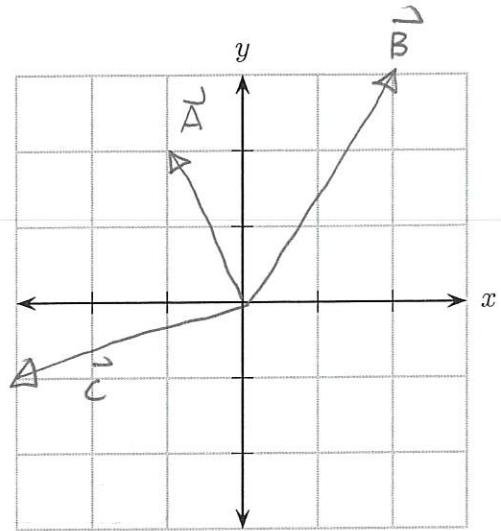
$$\Delta t = 2.48 \text{ s}$$

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Question 5

Let $\vec{A} = -1\hat{i} + 2\hat{j}$ and $\vec{B} = 2\hat{i} + 3\hat{j}$. Let $\vec{C} = \vec{A} - \vec{B}$. Draw \vec{A} , \vec{B} , and \vec{C} as accurately as possible on the grid provided and determine the magnitude of \vec{C} .

+4
+5



+3

$$\vec{C} = \vec{A} - \vec{B}$$

$$= -1\hat{i} + 2\hat{j} - (2\hat{i} + 3\hat{j})$$

$$= -3\hat{i} - \hat{j}$$

(+2)

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-3)^2 + 1^2}$$

$$= \sqrt{10} = 3.2$$

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Question 6

A red ball and a blue ball are launched horizontally from the same height above a horizontal floor. The red ball is launched with speed five times that of the blue ball. Let Δt_{red} be the time taken for the red ball to hit the floor and Δt_{blue} that for the blue ball to hit the floor. Ignoring air resistance, which of the following (choose one) is true?

- i) $\Delta t_{\text{red}} = \frac{1}{5} \Delta t_{\text{blue}}$.
- ii) $\frac{1}{5} \Delta t_{\text{blue}} < \Delta t_{\text{red}} < \Delta t_{\text{blue}}$.
- iii) $\Delta t_{\text{red}} = \Delta t_{\text{blue}}$.
- iv) $\Delta t_{\text{blue}} < \Delta t_{\text{red}} < 5 \Delta t_{\text{blue}}$.
- v) $\Delta t_{\text{red}} = 5 \Delta t_{\text{blue}}$.
- vi) $\Delta t_{\text{red}} > 5 \Delta t_{\text{blue}}$.

$y_i = h$ $y_f = 0$
 $v_{iy} = 0 \text{ m/s}$

$y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$
 $0 = h - \frac{g}{2} \Delta t^2$

$\Delta t = \sqrt{\frac{2h}{g}}$

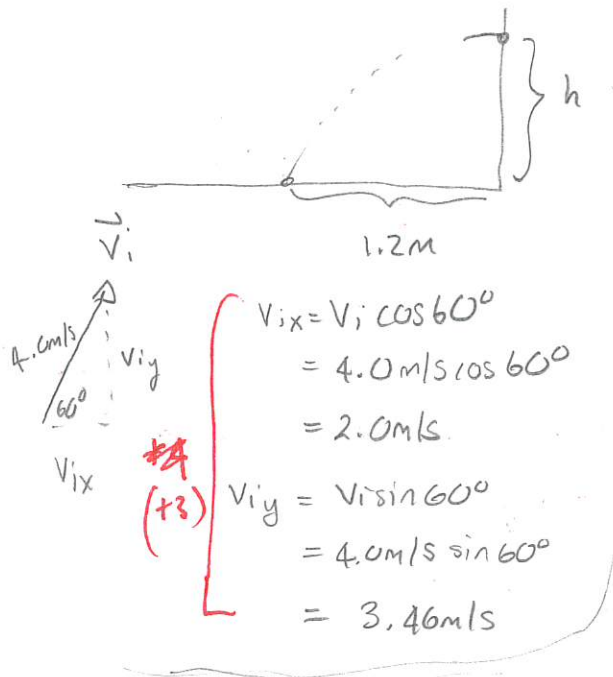
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4 only depends on h

Question 7

A bug sits on a horizontal floor and launches itself with a speed of 4.0 m/s at an angle of 60° above the horizontal. It lands on the vertical side of a box which is a horizontal distance of 1.2 m from when it launched.

- a) Determine the height (above the floor) of the location on the box where the bug lands.



$$\begin{aligned}
 V_{ix} &= V_i \cos 60^\circ \\
 &= 4.0 \text{ m/s} \cos 60^\circ \\
 &= 2.0 \text{ m/s} \\
 V_{iy} &= V_i \sin 60^\circ \\
 &= 4.0 \text{ m/s} \sin 60^\circ \\
 &= 3.46 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 t_i &= 0 \text{ s} & t_f &= \\
 x_i &= 0 \text{ m} & x_f &= 1.2 \text{ m} \\
 y_i &= 0 \text{ m} & y_f &= \\
 V_{ix} &= 2.0 \text{ m/s} & & \\
 V_{iy} &= 3.46 \text{ m/s} & & \\
 & & (+1) & \left[\begin{aligned} a_x &= 0 \text{ m/s}^2 \\ a_y &= -9.8 \text{ m/s}^2 \end{aligned} \right.
 \end{aligned}$$

Need time to travel

$$x_f = x_i + V_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2 \quad (+1)$$

$$1.2 \text{ m} = 0 \text{ m} + 2.0 \text{ m/s} \Delta t$$

$$\Rightarrow \Delta t = 0.60 \text{ s}$$

$$\left. \begin{aligned} & \\ & \end{aligned} \right\} (+2)$$

Then vertical distance traveled

$$y_f = y_i + V_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2 \quad (+1)$$

$$= 0 \text{ m} + 3.46 \text{ m/s} \times 0.60 \text{ s} - \frac{9.8 \text{ m/s}^2 (0.60 \text{ s})^2}{2}$$

$$y_f = 0.31 \text{ m}$$

$$\left. \begin{aligned} & \\ & \end{aligned} \right\} (+2)$$

Question 7 continued ...

- b) Does the bug pass the highest point of its trajectory before it lands on the box (i.e. is returning back to the floor or is still rising when it lands on the box)? Explain your answer.

When does it reach highest point? $v_{fy} = 0 \text{ m/s}$ (+1)

$$v_{fy} = v_{iy} + a_y \Delta t \quad (+1)$$

$$0 \text{ m/s} = 3.46 \text{ m/s} - 9.80 \text{ m/s}^2 \Delta t$$

$$\Rightarrow \Delta t = \frac{3.46 \text{ m/s}}{9.80 \text{ m/s}^2} = 0.35 \text{ s}$$

(+2)

This is before it hits the wall. So it is returning. (+2)

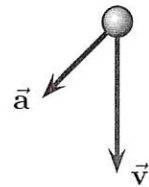
Find highest pt larger than landing is inconclusive. Could be



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Question 8

A ball moves on a flat horizontal surface. Viewed from above, the velocity and acceleration at an initial instant are as illustrated. Describe whether the ball's speed increases, decreases or stay constant in the instants that follow the initial instant. Does the ball's trajectory curve left, curve right or remain straight in the instants that follow? Explain your answers.



$$\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$$



\vec{v}_f is larger than $\vec{v}_i \Rightarrow$ speed increases +1

\vec{v}_f angles left \Rightarrow curves left +1

(+4)

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