Fundamental Mechanics: Class Exam 2

31 March 2023

Name: _____ **Total:** /70

Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$g = 9.80\,\text{m/s}^2$

Question 1 ✓

A 4.0 kg crate can slide along a frictionless horizontal surface. At one instant it moves *left* with speed 12.0 m/s. At another instant, 3.0 s later, it moves *left* with speed 3.0 m/s. During the period between these instants a rope pulls horizontally with a constant force on the crate. Determine the tension in the rope and describe the direction in which the rope pulls.

3.0 m/s ⓑ ← 12.0 m/s ⓑ

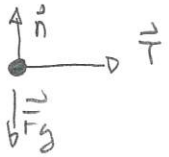
Acceleration

+1 $v_f = v_i + a\Delta t$

+3 $-3.0\,\text{m/s} = -12.0\,\text{m/s} + a\,3.0\,\text{s}$

$9.0\,\text{m/s} = 3.0\,\text{s}\,a$

$\Rightarrow a = 3.0\,\text{m/s}^2$



$\Sigma F_x = ma_x$

$\Rightarrow T = ma$

$= 4.0\,\text{kg} \times 3.0\,\text{m/s}^2 = 12\,\text{N}$ +3

Acceleration right ⇒ net force right

must actually say +1

/~~XX~~ 8

Question 2 ✓

A ball with mass m is attached to stretchy rubber band. The ball is dropped from rest. The ball falls downward but it eventually slows as it reaches its lowest point. While it slows on its way down, which of the following (choose one) is true regarding the tension, T, in the string?

i) $T = mg$

ii) $T < mg$

iii) $T > mg$

$\Sigma F_y = ma_y$ accel is ↑

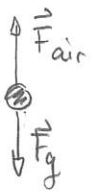
$\Rightarrow T > mg$

/~~XX~~ 4

Question 3 ✓

A 0.040 kg ball is initially at rest at the bottom of a frictionless vertical tube of length 1.2 m. The ball is pushed by air which exerts a constant upward force of 1.5 N on the ball. Determine the velocity of the ball when it reaches the top of the tube.

+1



+1 $\Sigma F_y = ma_y$

$F_{air} - mg = ma_y$

$\Rightarrow 1.5N - 0.040kg \times 9.8 m/s^2 = 0.040kg\, a$

$\Rightarrow 1.1N = 0.040kg\, a \Rightarrow a = 27.7 m/s^2$ +4

$y_0 = 0$ $y_f = 1.2m$

$v_{yi} = 0 m/s$ $v_{yf} = ?$

$v_{yf}^2 = v_{iy}^2 + 2a\Delta y$ +1

+3

$v_{yf}^2 = (0 m/s)^2 + 2 \times 27.7 m/s^2 \times 1.2m$

$v_{yf}^2 = 66.5 m^2/s^2$

$v_{yf} = \sqrt{66.5 m^2/s^2}$

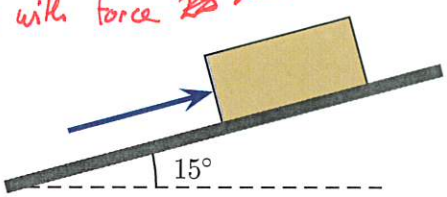
$v_{yf} = 8.2 m/s$

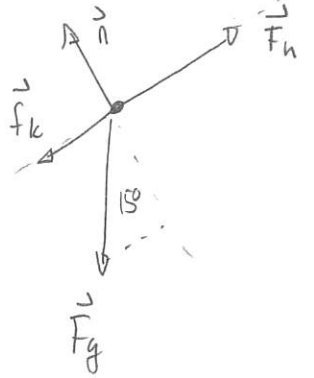
/~~XX~~ 10

Question 4

A 50 kg box lies on a ramp inclined at an angle of 15° above the horizontal. The coefficient of kinetic friction is 0.16. You push against the block parallel to the ramp and uphill. Determine the acceleration of the box provided that it moves up the ramp.

with force ~~25~~ 300N





+2

$\Sigma F_x = ma_x$

$\Sigma F_y = ma_y = 0$

+1

$F_g = mg$

$f_k = \mu_k n$

+2

	x	y
$\vec{n}$	0	n
$\vec{F}_h$	F_h	0
$\vec{f}_k$	$-\mu_k n$	0
$\vec{F}_g$	$-mg\sin 15^\circ$	$-mg\cos 15^\circ$

+2

+3

$\Sigma F_x = ma_x$

$\Rightarrow F_h - \mu_k n - mg\sin 15^\circ = ma_x$ +1

$\Sigma F_y = 0$

$\Rightarrow n - mg\cos 15^\circ = 0$

$\Rightarrow n = mg\cos 15^\circ$ +1

$F_h - \mu_k mg\cos 15^\circ - mg\sin 15^\circ = ma_x$

$F_h - mg\left[\mu_k \cos 15^\circ + \sin 15^\circ\right] = ma_x$

$300N - 50kg \times 9.8 m/s^2 \left[0.16\cos 15^\circ + \sin 15^\circ\right] = 50kg\, a_x$

$97.5N = 50kg\, a_x$

$a_x = \frac{97.5N}{50kg} = 1.9 m/s^2$

+2

/~~XX~~ 14

Question 5

Two carts are in contact while on a horizontal frictionless surface. Cart A has a smaller mass than cart B. Someone pushes horizontally to the right on cart A with force 50 N. Which of the following (choose one) is true?

i) The force that A exerts on B is less than 50 N.
ii) The force that A exerts on B is more than 50 N.
iii) The force that A exerts on B is exactly 50 N.

Explain your answer.

Both carts accelerate to right since the net force on the pair is right.

For cart A

$\Sigma F_x = ma_x > 0 \Rightarrow F_{B\text{ on }A} < 50N$

Then $F_{A\text{ on }B} = F_{B\text{ on }A}$

$\Rightarrow F_{A\text{ on }B} < 50N$

/~~XX~~ 6

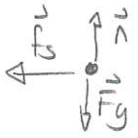
Question 6

A box rests on a delivery cart on a horizontal floor. The cart is initially at rest and is pulled, moving to the left with constantly increasing speed. While this happens, the box does not slip relative to the cart. Ignore air resistance. Which of the following (choose one) is true while the cart speeds up to the left?

i) There is no friction force on the box.
ii) There is a static friction force on the box to the right.
iii) There is a static friction force on the box to the left.
iv) There is a kinetic friction force on the box to the right.
v) There is a kinetic friction force on the box to the left.

Explain your answer.

Acceleration is left. Thus net force on box is left. For box this means friction is left



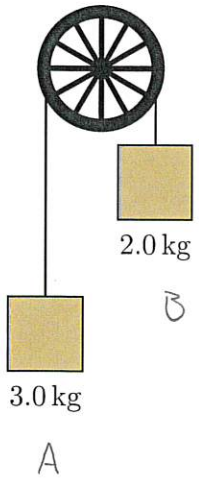
It is static since box does not slip

only static +1
only left +1
both +2

/~~XX~~ 6

Question 7

Two blocks are connected by a massless string that runs over a massless pulley. Ignore air resistance and friction. Determine the acceleration of the block on the right. *Show the entire argument and derivation starting with Newton's Second Law.*

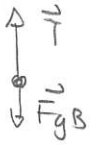


For block A

(+1)

For block B

(+1)



Let a be the magnitude of acceleration.

Block A

$\sum F_y = m_A a_y$

$T - m_A g = m_A(-a)$ ← negative (+3)

(+1)

$T - m_A g = -m_A a$

$T = m_A g - m_A a$

Block B

$\sum F_y = m_B a_y$] at least one of these. (+1)

$T - m_B g = m_B a$

(+1)

$m_A g - m_A a - m_B g = m_B a$

$(m_A - m_B) g = m_A a + m_B a$

$= (m_A + m_B) a$

(+6)

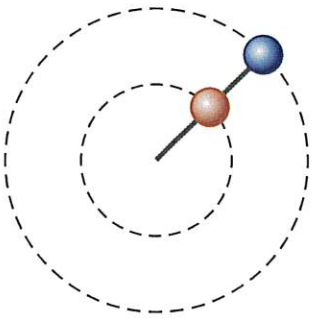
$\Rightarrow a = \frac{m_A - m_B}{m_A + m_B} g = \frac{3.0\,kg - 2.0\,kg}{5.0\,kg} 9.80\,m/s^2$

$a = 1.96\,m/s^2$ up ↑

/~~XX~~ 14

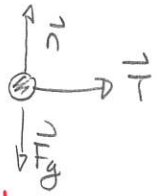
Question 8

Two 3.0 kg balls are attached by strings and each moves on a frictionless horizontal surface in a circle, The outer ball follows a path with radius 4.0 m and the inner ball one with radius 2.0 m. The diagram shows this viewed from above. The balls each take 0.25 s to complete one circle. Determine the tension in the outer string.



On outer ball

left side



(+0.5)

$\Sigma F_x = ma_x$ (+1)

$T = m\frac{v^2}{r}$ (+2)

include F_g (-2.5)

Now $v = \frac{2\pi r}{t}$ where t is time for one orbit (+1) actual numbers (+1)

$T = m\frac{4\pi^2 r^2}{t^2 r} = m\frac{4\pi^2 r}{t^2}$

$T = 3.0\,kg \frac{4\pi^2 \times 4.0\,m}{(0.25\,s)^2}$ (+2.5)

$= 7600\,N$

/~~XX~~ 8