Quantum Theory I: Homework 23

Due: 5 May 2023

1 Distinguishing between angular momentum states

Two experimentalists, Alice and Bob, each have an ensemble of identical and distinguishable particles, each in an identical spherically symmetric potential. Both ensembles are prepared so that each particle is in the state

$$|\psi\rangle = |1, 0\rangle$$

where $|l, m\rangle$ denotes a standard eigenstate of total angular momentum and the z-component of angular momentum. Alice measures the x-component of the angular momentum of each particle in *her* ensemble while Bob measures the z-component of the angular momentum of each particle in *his* ensemble.

a) Determine the expectation value, $\langle L_z \rangle$, for Bob's measurements.
b) Determine the uncertainty, ΔL_z, for Bob's measurements.
c) Determine the expectation value, $\langle L_x \rangle$, for Alice's measurements.
d) Determine the uncertainty, ΔL_x, for Alice's measurements.
e) Suppose that Alice or Bob are each given one more particle in the state $|\psi\rangle = |1, 0\rangle$ and each measures the same component of angular momentum that they had previously. Will either Alice or Bob obtain one particular measurement outcome with 100% certainty? If so who does and what is the outcome? Justify your answer.

2 Measurements of L_x and L_y

A particle is in a central potential; the standard angular momentum eigenstates are denoted $|l, m\rangle$.

a) Consider the states

$$\begin{aligned}
|\phi_1\rangle &:= \frac{1}{2}\left\{|1,1\rangle + \sqrt{2}\,|1,0\rangle + |1,-1\rangle\right\} \\
|\phi_2\rangle &:= \frac{1}{\sqrt{2}}\left\{|1,1\rangle - |1,-1\rangle\right\} \\
|\phi_3\rangle &:= \frac{1}{2}\left\{|1,1\rangle - \sqrt{2}\,|1,0\rangle + |1,-1\rangle\right\}.
\end{aligned}$$

Show that each of these states is an eigenstate of both $\hat{L}_x$ and $\hat{\mathbf{L}}^2$ and show that each is normalized. List the eigenvalues and the possible measurement outcomes for L_x and $\mathbf{L}^2$ for each state.

b) A particle is in the state

$$|\Psi\rangle := \frac{1}{\sqrt{2}}\Big\{|1,1\rangle + |1,-1\rangle\Big\}$$

Show that this is an eigenstate of $\hat{L}_y$ and $\hat{\mathbf{L}}^2$ and list the eigenvalues. Provide a physical interpretation of this state in terms of outcomes of measurements of L_y and $\mathbf{L}^2$.

c) A particle is in the state

$$|\Psi\rangle := \frac{1}{\sqrt{2}}\Big\{|1,1\rangle + |1,-1\rangle\Big\}$$

and is subjected to a measurement of L_x. List the possible outcomes of this measurement and the probabilities with which they occur.

3 Angular momentum measurements and kets

Consider a particle in the state $|l = 1, m = -1\rangle$. Use the operator and ket formalism to answer the following.

a) Determine the expectation values of $\hat{L}_x, \hat{L}_y$ and $\hat{L}_z$.

b) Determine the uncertainties $\Delta L_x, \Delta L_y$ and ΔL_z.

c) Is there any component of angular momentum, whose measurement outcome is one value with 100% certainty? If so what is it and what is the measurement outcome?