Quantum Theory I: Homework 22

Due: 2 May 2023

1 Truncated radial momentum states

Consider the state that corresponds to the wavefunction

$$\Psi(r,\theta,\phi) = \begin{cases} \dfrac{A}{r}\, e^{i\alpha r/\hbar} & \text{if } r \leqslant R \\ 0 & \text{if } r > R \end{cases}$$

where R is a constant with units of meters.

a) Determine the normalization constant A (don't forget the angular coordinates in this process).

b) Suppose you have many copies of the particle in the same state and measure the distance from the origin for each. Predict what the average of the outcomes should be.

2 Angular momentum squared for a spin-1/2 particle

The general angular momentum squared operator is

$$\hat{\boldsymbol{J}}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2.$$

Consider this for a spin-1/2 particle. In this case, $J_x = S_x$, etc.

a) Determine the matrix that represents $\hat{\boldsymbol{S}}^2$.

b) Suppose that $\boldsymbol{S}^2$ were measured for a spin-1/2 particle. What could you predict about the outcome? Would this depend on the state of the spin-1/2 particle?

c) Based on this, how would you describe $|\boldsymbol{S}|$?

d) Is it possible that for such a particle $S_z = \pm|\boldsymbol{S}|$? Explain your answer.