Quantum Theory I: Homework 20

Due: 18 April 2023

1 Position space wavefunctions for the harmonic oscillator

In the following let $\psi_n(x) := \langle x|n\rangle$ where $|n\rangle$ is the n^{th} energy eigenstate for the harmonic oscillator. Note that

$$\psi_1(x) = \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4} x\, e^{-m\omega x^2/2\hbar}.$$

a) Starting with the wavefunction $\psi_1(x)$, show that

$$\psi_2(x) = \left(\frac{m\omega}{4\pi\hbar}\right)^{1/4} \left(2\frac{m\omega}{\hbar}x^2 - 1\right) e^{-m\omega x^2/2\hbar}.$$

b) Use the position space wavefunctions to show explicitly that $\langle 0|2\rangle = 0$.

2 Position and momentum expectation values for an harmonic oscillator

An ensemble of identical harmonic oscillators, each of mass m and with frequency ω, are in the state

$$|\Psi\rangle = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle.$$

a) If the position of each oscillator were to be measured, what would one expect for the average of the outcomes?

b) If the momentum of each oscillator were to be measured, what would one expect for the average of the outcomes?

3 Harmonic oscillator temporal evolution

Consider an ensemble of harmonic oscillators, all of which are in one of the following states at $t = 0$:

$$|\Psi_A(0)\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$$
$$|\Psi_B(0)\rangle = \frac{5}{13}|1\rangle + \frac{12}{13}|2\rangle$$
$$|\Psi_C(0)\rangle = \frac{1}{\sqrt{2}}|3\rangle + \frac{1}{\sqrt{2}}|4\rangle$$
$$|\Psi_D(0)\rangle = \frac{1}{\sqrt{2}}|3\rangle + \frac{1}{\sqrt{2}}|5\rangle$$

In each case the expectation value of position could be measured.

a) Rank these in order of increasing frequency of oscillation of $\langle x\rangle$.

b) Rank these in order of increasing amplitude of oscillation of $\langle x\rangle$.