

Final Review

Final

<p>When your class meets regularly</p> <p>Section 001 10am</p> <p>Section 002 11am</p>	<p>When the final happens</p> <p>Mon, May 15, 10-11:50am</p> <p>Weds, May 17, 10-11:50am</p>
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Covers: Entire Semester

Bring: * Calculator - note about using mine

* Total four 3" x 5" single-sided cards (or equiv surface area)

Review: Previous finals 2018 } all questions.
2022 }

Note: version 2 only has solutions for questions different to version 1

This review: only covers Ch 10, 11, 12, 13

Chapters 10-12

<p>Equations</p> <p>$w = w_0 + \alpha t$</p> <p>$\theta = \theta_0 + w_0 t + \frac{1}{2} \alpha t^2$</p> <p>$w^2 = w_0^2 + 2 \alpha \Delta \theta$</p> <p>$v_t = w r$</p> <p>$a_t = \alpha r$</p>	}	<p>$I = \sum m_i r_i^2$</p> <p>others given</p> <p>$\tau = \vec{r} \times \vec{F}$</p> <p>$= r F \sin \phi$</p> <p>$\tau = I \alpha$</p>	}	<p>vector cross product rules</p> <p>$K_{rot} = \frac{1}{2} I \omega^2$</p> <p>$\vec{L} = I \vec{\omega}$</p>
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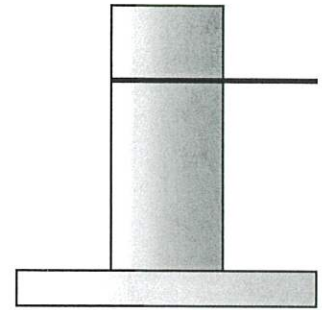
Quiz 1 → typo

→ 40% ~ better

→ 40% ~ 60%

311 Flywheel and axle

A particular flywheel is a solid disk with mass 0.150 kg and radius 0.075 m. This is mounted to an 0.400 kg axle which is a hollow cylinder with radius 0.020 m. The entire arrangement is initially at rest and is subsequently pulled with constant tension by a string that is wound around the axle. It reaches an angular velocity of 40 rad/s in 5.0 s. A side view is illustrated. Determine the tension in the string. (131Sp2023)



Answer:

Rotational kinematics \rightarrow angular acceleration

Rotational dynamics \rightarrow torque \rightarrow tension in string.

Angular acceleration:

$$\omega = \omega_0 + \alpha t$$

$$40 \text{ rad/s} = 0 \text{ rad/s} + \alpha \cdot 5.0 \text{ s}$$

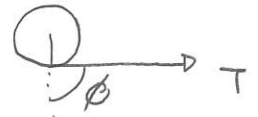
$$\Rightarrow \alpha = 8.0 \text{ rad/s}^2$$

$$\omega_0 = 0 \text{ rad/s}$$

$$\omega = 40 \text{ rad/s} \quad t = 5.0 \text{ s}$$

Rotational dynamics $\tau_{\text{net}} = I\alpha$

$$\tau_{\text{net}} = \tau_{\text{string}} + \tau_{\text{grav}} + \tau_{\text{axle}} = 0 \quad \text{since } r=0$$



$$\tau_{\text{string}} = rT \sin \phi = r_{\text{cyl}} T \sin 90^\circ = rT$$

$$\Rightarrow r_{\text{cyl}} T = I\alpha$$

$$T = \frac{I\alpha}{r_{\text{cyl}}}$$

$$\text{But } I = I_{\text{cylinder}} + I_{\text{disk}} = M_{\text{cyl}} r_{\text{cyl}}^2 + \frac{1}{2} M_{\text{disk}} r_{\text{disk}}^2$$

$$= 0.400 \text{ kg} \times (0.020 \text{ m})^2 + \frac{1}{2} \times 0.150 \text{ kg} \times (0.075 \text{ m})^2$$

$$= 1.6 \times 10^{-4} \text{ kg m}^2 + 4.2 \times 10^{-4} \text{ kg m}^2 = 5.8 \times 10^{-4} \text{ kg m}^2$$

$$T = \frac{5.8 \times 10^{-4} \text{ kg m}^2 \times 8.0 \text{ rad/s}^2}{0.020 \text{ m}} = 0.23 \text{ N}$$

Quiz 2 80% - 90% \approx 90%

To analyze this note that energy is conserved.

$$E_f = E_i$$

$$K_{rot f} + K_{trans f} + U_{gf} = \cancel{K_{rot i}} + \cancel{K_{trans i}} + U_{gi}$$

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 = U_{gi} - U_{gh} = mgh$$

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 = mgh$$

For the slipping ball $\omega_f = 0 \Rightarrow v_f^2 = 2gh \Rightarrow v_f = \sqrt{2gh}$ slipping

For the rotating ball $\omega_f = v_f/R \Rightarrow \frac{1}{2} M v_f^2 + \frac{1}{2} \frac{I}{R^2} v_f^2 = mgh$

$$\Rightarrow v_f^2 + \frac{I}{MR^2} v_f^2 = 2gh$$

$$\Rightarrow v_f^2 (1 + I/MR^2) = 2gh$$

$$\Rightarrow v_f = \sqrt{2gh / (1 + I/MR^2)}$$
 rolling

$1 + I/MR^2$ is larger than 1 $\Rightarrow v_f$ rolling $<$ v_f slipping

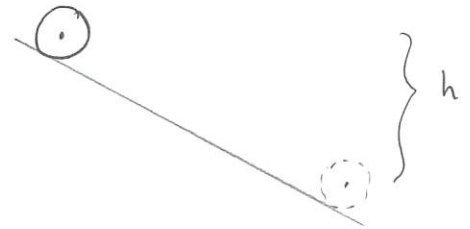
Quiz 3 30% - 90% \approx 60% \rightarrow 80%

Chapter B

$$F = G \frac{M_1 M_2}{r^2} \quad U = -G \frac{M_1 M_2}{r}$$

How to: use Newton's Second Law + Univ law of gravity to derive results about

- * gravitational acceleration
- * orbital motion



need

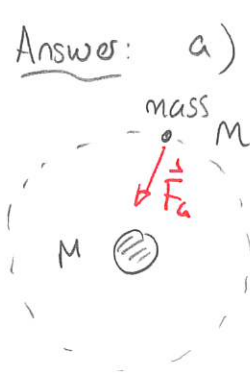
B = C < A

346 Orbiting satellite

A satellite orbits a planet with mass M_p in a circular orbit with radius r . The satellite's speed is constant. (131Sp2023)

- Starting with and using Newton's Second Law, derive an expression for the satellite's speed in terms of M_p and r .
- Determine the speed of a satellite in a uniform circular orbit 60000 m above the surface of the dwarf planet Ceres (mass 9.4×10^{20} kg and radius 4.7×10^5 m).

Answer: a)



$F = ma$ ← essential
 and $a = v^2/r$

$F_g = G \frac{M m_p}{r^2} = m a$

$G \frac{M_p}{r^2} = \frac{v^2}{r}$

$\Rightarrow v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$

b) $r = 4.7 \times 10^5 \text{ m} + 6.0 \times 10^4 \text{ m} = 5.3 \times 10^5 \text{ m}$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 9.4 \times 10^{20} \text{ kg}}{5.3 \times 10^5 \text{ m}}}$$

$= 344 \text{ m/s}$



Cause Evals: