Phys 131 2023

Lecture 40

Tues: Warm Up 15

Diagnostic Test

- extra credit details in email

Thurs: Discussion/quiz

Ex:

Newton's Universal Law of Gravitation

Newtons universal law of gravitation gives:

The gravitational force exerted by an object with mass m_1 on an object with mass m_2 is:

$$F = G\frac{m_1 m_2}{r^2}$$

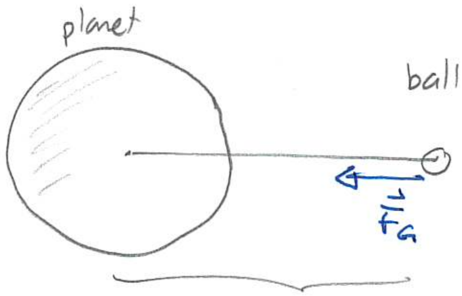
where r = distance between the objects

$$G = 6.67 \times 10^{-11} \frac{N\,m^2}{kg^2}$$

Quiz 1 80% – 100% } 60% – 90%

Acceleration near to a planet

Consider a ball near to a planet. Let M_p be the planet mass and M_b be the ball mass. The gravitational force exerted by the planet requires the radius from the center of the planet to the center of the ball



$r \longrightarrow$ used in Newton's Law of gravitation.

Quiz #2 20% – 30% $\lesssim$ 40%

$$\vec{F}_{net} = M_{ball}\,\vec{a}_{ball} \Rightarrow \vec{F}_G = M_{ball}\,\vec{a}_{ball}$$

$$\Rightarrow G\frac{\cancel{M_{ball}}\,M_p}{r^2} = \cancel{M_{ball}}\,a_{ball}$$

$$\Rightarrow a = G\frac{M_p}{r^2}$$

The acceleration does not depend on the mass of the falling object.

Note that this allows one to determine the mass of the planet

Determine radius of planet R_p

Determine free-fall acceleration of object near planet's surface, a

$\longrightarrow$ $a = G\dfrac{M_p}{R_p^2}$ determines mass of planet

340 Planet mass and free fall acceleration.

The radius of Earth can be obtained by geometrical measurements and is, on average 6.371×10^6 m. The acceleration due to Earth's gravity at the surface is $9.81\,\mathrm{m/s^2}$. *(131Sp2023)*

a) Determine the mass of Earth and compare your result to one that you could look up.

b) Grand Junction is located approximately 1400 m above sea level. Determine the free fall acceleration due to Earth's gravity at Grand Junction. Does this differ much from the acceleration at sea-level?

Answer: a) $$a = g = G\frac{M_p}{R_p^2}$$

$$\Rightarrow M_p = \frac{g R_p^2}{G} = \frac{9.81\,\mathrm{m/s^2} \times (6.371\times 10^6\,\mathrm{m})^2}{6.67\times 10^{-11}\,\mathrm{Nm^2/kg^2}}$$

$$= 5.97\times 10^{24}\,\mathrm{kg}$$

(5.98×10^{24} kg according to PhysLink)

b) Here $r = R_p + h$ where h is Grand Junction's altitude.

$$a = G\frac{M_p}{(R_p+h)^2} = 6.67\times10^{-11}\,\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}\;\frac{5.97\times10^{24}\,\mathrm{kg}}{[(6.371 + 0.0014\,\mathrm{m})\times 10^6]^2}$$

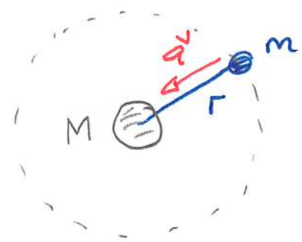
$$= 9.81\,\mathrm{m/s^2}$$

There is no noticeable difference (to three significant figures)

Thus acceleration due to Earth's gravity is roughly constant even at distances of ~1500m above sea level.

Orbital motion

Gravity can describe the orbit of objects. Consider an object that orbits a planet in a circle. Let r be the radius of the orbit. Can an attractive gravitational force describe this orbit?



Demo: Newton's Cannon.

The object has

Radially inward acceleration $\Rightarrow$ Net force is inward $\leftarrow$ Gravitational force provides this.

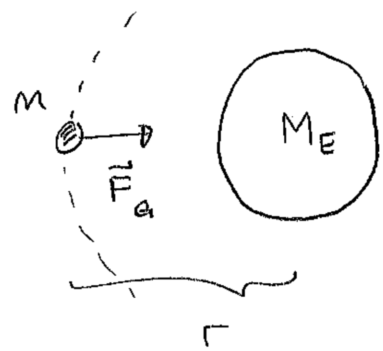
343 Newton's cannonball

Newton's cannonball was a thought experiment in which a cannonball is fired from the top of a mountain and eventually orbits Earth. Suppose that a cannonball with mass m is fired horizontally from a mountain with altitude h above sea level, which is distance R_E from Earth's center. *(131Sp2023)*

a) **Starting with and using** Newton's Second Law, determine an expression for the cannonball's launch velocity, v so that it follows a circular orbit at height h above Earth's sea level.

b) Determine the speed with which the cannonball should be launched from the altitude of 8000 m above sea level to orbit as described above.

Answer



$$\vec{F} = m\vec{a}$$

$$\Rightarrow F_G = ma = \frac{mv^2}{r}$$

$$\Rightarrow G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\Rightarrow v^2 = \frac{GM_E}{r}$$

Here $r = R_E + h$

R_E radius Earth

$$\Rightarrow v^2 = \frac{GM_E}{R_E + h}$$

$$\Rightarrow v = \sqrt{\frac{GM_E}{R_E + h}}$$

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b) $$v = \frac{6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \times 5.98\times10^{24}\,\text{kg}}{(6.371 + 0.008)\times10^{6}\,\text{m}} = 7900\,\text{m/s}$$

(speed of sound 330 m/s)

Note that this relates to the period of orbit via:

$$v = \frac{2\pi r}{T} \quad \text{period}$$

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \quad \Rightarrow \quad \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\Rightarrow \quad T^2 = \underbrace{\left(\frac{4\pi^2}{GM}\right)}_{\text{constant.}} r^3$$

The rule T^2 is proportional to r^3 was known before Newton's theories. These eventually explained the rule.