Phys 131 2023

Lecture 39

Mon: HW by 5pm

Ex 325, 328, 330, 331, 332, 334, 335, 336

Tues: Redo diagnostic

Warm Up 15

Angular momentum

There is a rotational analog of the conservation of momentum.

Define:

> The angular momentum of a rotating object about a fixed axle is
>
> $$\vec{L} = I\vec{\omega}$$
>
> units: $kg\ m^2/s$
>
> vector
>
> where I = moment of inertia about the axle
>
> $\vec{\omega}$ = angular velocity about the axle

Newton's Second Law gives:

> The angular momentum of any system satisfies.
>
> $$\vec{\tau}_{\substack{net \\ external}} = \frac{d\vec{L}}{dt}.$$

Demo: Bicycle wheel / gyroscope.

Then

> If the net external torque is zero, the angular momentum of the system is constant.

Conservation of angular momentum

Quiz! 20% – 80% $\lesssim$ 20%

333 Rotating disk and hoop

A disk with mass M and radius R rotates in a horizontal plane with constant angular velocity, ω_i. A hoop with mass M and radius R is gently lowered onto the disk so that the center of the hoop coincides with the center of the disk. The hoop sticks to the disk and the two rotate with angular velocity ω_f. Which of the following is true? *(131Sp2023)*

i) $\omega_f = \omega_i$

ii) $\omega_f = \dfrac{1}{2}\,\omega_i$

iii) $\omega_f = \dfrac{1}{\sqrt{2}}\,\omega_i$

iv) $\omega_f = \dfrac{1}{3}\,\omega_i$

v) $\omega_f = \dfrac{1}{\sqrt{3}}\,\omega_i$

Explain your answer.

Answer: Angular momentum is conserved $\quad L = I\omega$

Before

$\omega_{hoop\,i} = 0$

$\omega_{disk\,i} = \omega_i$

After

$\left.\begin{array}{l}\omega_{hoop\,f} = \omega_f \\ \omega_{disk\,f} = \omega_f\end{array}\right\}$ same.

$$L_f = L_i$$

$$L_{f\,hoop} + L_{f\,disk} = \cancel{L_{f\,hoop\,i}} + L_{disk\,i}$$

$$I_{hoop}\,\omega_f + I_{disk}\,\omega_f = I_{disk}\,\omega_i$$

$$\left(MR^2 + \tfrac{1}{2}MR^2\right)\omega_f = \tfrac{1}{2}MR^2\,\omega_i$$

$$\tfrac{3}{2}\cancel{MR^2}\,\omega_f = \tfrac{1}{2}\cancel{MR^2}\,\omega_i \Rightarrow \omega_i = 3\omega_f \Rightarrow \omega_f = \tfrac{1}{3}\omega_i$$

Demo: Rotating platform.

Gravitational forces

Newton's mechanics provides a general framework:

Specify forces

$\longrightarrow$ Find acceleration from Newton's second law

$$\vec{F}_{net} = \Sigma \vec{F} = m\vec{a}$$

$\longrightarrow$ Relate to velocity, position

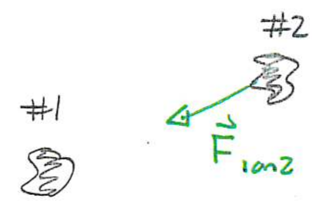
$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

We still need general rules for describing particular forces, using first principles. Two examples are:

1) gravitational forces
2) electric + magnetic forces Phys 132

Newton proposed a general rule for gravitational forces. The universal law of gravitation is

> Every object with non-zero mass exerts a gravitational force on every other object with non-zero mass.
> The force
>
> 1) is attractive along the line between the objects
>
> 
>
>
> 2) has magnitude
>
> $$F_{1on2} = G\,\frac{m_1 m_2}{r^2}$$
>
> where r is the distance between the two objects.

The quantity G is the same for any pair of objects and is called the universal gravitation constant. It's value determined by Cavendish is

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

337 Ordinary objects and gravitational forces

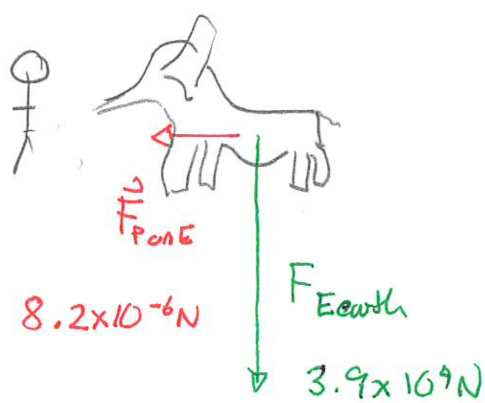
A 100 kg person stands near 4000 kg elephant. Their centers of mass are 1.8 m apart. Determine the gravitational force exerted by the person on the elephant. *(131Sp2023)*

$$F_{P\,on\,E} = G\frac{M_P M_E}{r^2}$$

$$= 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\;\frac{100\,kg \times 4000\,kg}{(1.8\,m)^2}$$

$$= 8.2\times10^{-6}\,N$$

This is small compared to Earth's gravitational force



$$F_g = M_E g$$

$$= 4000\,kg \times 9.8\,m/s^2$$

$$= 3.9\times10^{4}N$$

Demo: Cavendish Balance YouTube Video

Quiz 2