Thurs: Discussion / quiz

Ex: 316, 317, 318, ~~322~~ 322, 323, 327

Fri: -

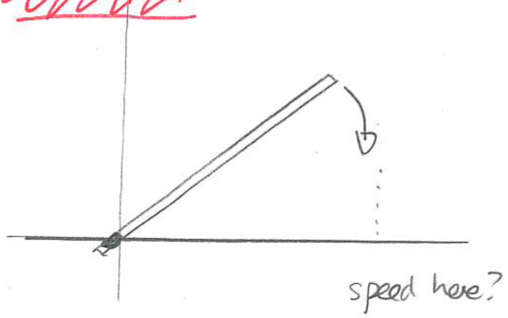
Mon: HW by 5pm

Rotational energy

In a situation where the pivot point is fixed, we can show that

$$E = K_{rot} + U_{grav} + U_{spring} + \dots$$

is constant provided that non-conservative forces do zero work. We apply this to a toppling rod.

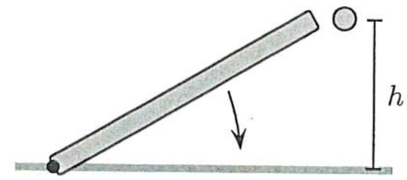


Demo: After: U Iowa Falling Chimney

MIT Video

319 Toppling rod versus freely falling ball, 1

A rigid rod with uniformly distributed mass M and length L can pivot about a frictionless axle in a horizontal surface. The rod is held at rest with one end height h above the surface. A ball with mass m is also held at rest alongside the tip of the rod. Both are released at the same time. *(131Sp2023)*



a) Determine the speed of the ball just before it hits the horizontal surface.

b) Determine the speed of the tip of the rod just before it hits the horizontal surface.

c) Which hits first?

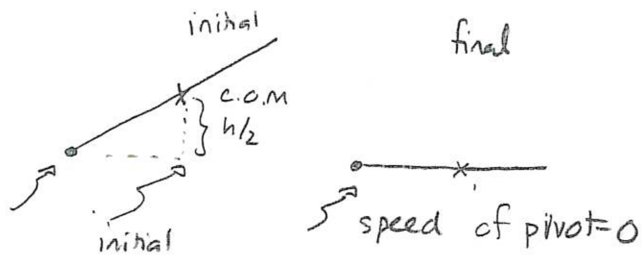
Answer a) $E_f = E_i$

$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2} m v_f^2 + \cancel{mgy_f} = \cancel{\frac{1}{2} m v_i^2} + mgy_i$

$\Rightarrow \frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g h \Rightarrow v_f^2 = 2gh$

$\Rightarrow \boxed{v_f = \sqrt{2gh}}$

b) Use the pivot as an axle



$E_f = E_i$

$K_{rotf} + K_{transf} + \cancel{U_{gf}} = \cancel{K_{roti}} + \cancel{K_{transi}} + U_{gi}$

$\frac{1}{2} I \omega_f^2 + \cancel{\frac{1}{2} M v_f^2} = M g \frac{h}{2}$

$\Rightarrow \omega_f = \frac{2v_f}{L}$

I about end $= \frac{1}{3} M L^2$

$\frac{1}{2} \frac{1}{3} \cancel{M} L^2 \omega_f^2 = \cancel{M} g \, h/2$

$\Rightarrow \frac{1}{3} L^2 \omega_f^2 = gh$

Now the speed of the tip is $v_f = L\omega_f$

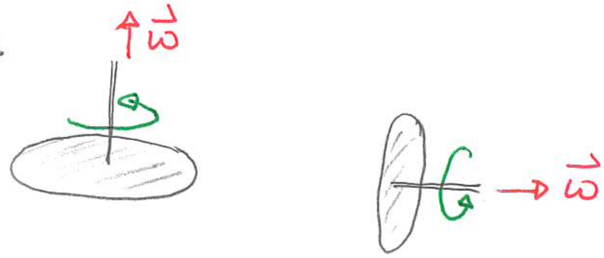
$\Rightarrow \frac{1}{3} v_f^2 = gh \Rightarrow v_f = \sqrt{3gh}$

Vectors and rotational motion

Objects can rotate about multiple different axes of rotation. and one can account for these via an angular velocity vector. The rule is:

> The angular velocity $\vec{\omega}$ of an object rotating about an axis is a vector with
>
> 1) magnitude $\omega = \left|\frac{d\theta}{dt}\right|$
>
> 2) direction - along axis using r.h.rule
>
> Fig 10.5

Thus



Then the angular acceleration is also a vector

$$\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}}$$

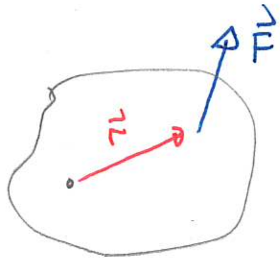
Thus the torque is also a vector so that

$$\vec{\tau}_{net} = I\vec{\alpha}$$

We need a method to construct a torque vector. This will combine the force vector $\vec{F}$ and a vector from the pivot. So we need to multiply

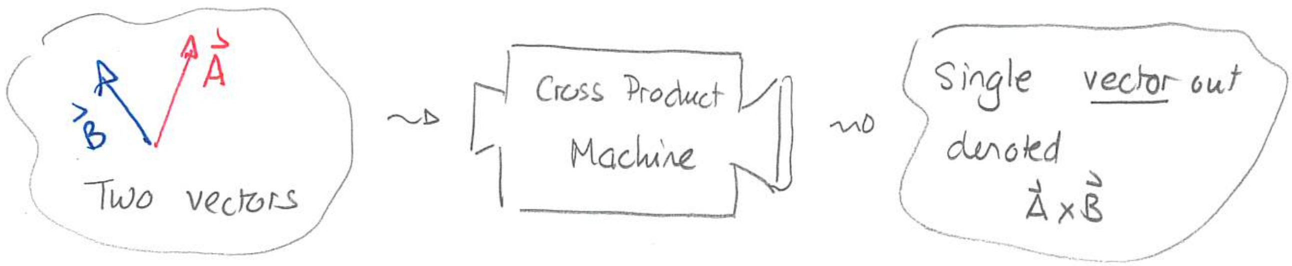
$\vec{r}$, $\vec{F}$

to produce a vector.



Vector cross product

The vector cross product takes two vectors and produces a third:



The two ways (equivalent) to calculate this are

Method 1 The cross product is a vector with

1) magnitude

$$AB \sin\phi$$

magnitude A — magnitude B — angle between $\vec{A}$ and $\vec{B}$

2) direction – perpendicular to both vectors
– sense given by r.h. rule. – curl fingers from $\vec{A}$ to $\vec{B}$

Quiz 1 70% – 90% $\{$40% – 80%

Method 2 Use algebra:

1) the cross product is linear in both arguments

2)
$$\hat{\imath}\times\hat{\imath}=0 \qquad \hat{\imath}\times\hat{\jmath}=\hat{k} \qquad \hat{\jmath}\times\hat{\imath}=-\hat{k}$$
$$\hat{\jmath}\times\hat{\jmath}=0 \qquad \hat{\jmath}\times\hat{k}=\hat{\imath} \qquad \hat{k}\times\hat{\jmath}=-\hat{\imath}$$
$$\hat{k}\times\hat{k}=0 \qquad \hat{k}\times\hat{\imath}=\hat{\jmath} \qquad \hat{\imath}\times\hat{k}=-\hat{\jmath}$$

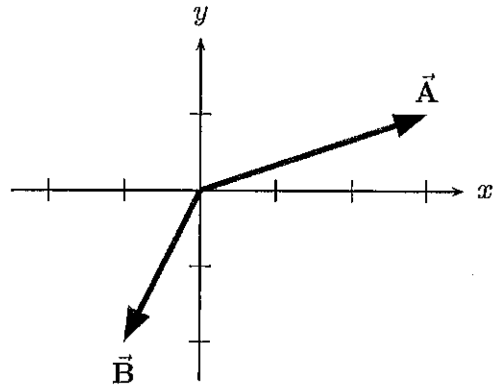
Warm Up 1

These satisfy:

1) $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$
2) $\vec{A}\times(\lambda\vec{B}) = \lambda(\vec{A}\times\vec{B})$
3) $\vec{A}\times\vec{A} = 0$
4) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

329 Vector cross product

Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. Determine $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$. *(131Sp2023)*



Answer:

$$\vec{A} = 3\hat{\imath} + \hat{\jmath}$$

$$\vec{B} = -\hat{\imath} - 2\hat{\jmath}$$

$$\vec{A} \times \vec{B} = (3\hat{\imath} + \hat{\jmath}) \times (-\hat{\imath} - 2\hat{\jmath})$$

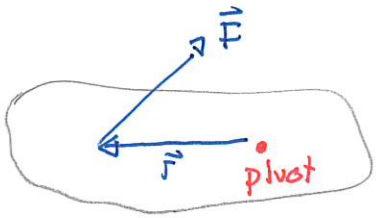
$$= -3\hat{\imath}\times\hat{\imath} - 3\hat{\imath}\times 2\hat{\jmath} - \hat{\jmath}\times\hat{\imath} - \hat{\jmath}\times 2\hat{\jmath}$$

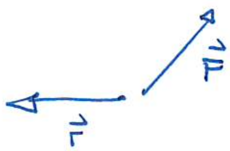
$$= -3\underbrace{\hat{\imath}\times\hat{\imath}}_{0} - 6\underbrace{\hat{\imath}\times\hat{\jmath}}_{\hat{k}} - \underbrace{\hat{\jmath}\times\hat{\imath}}_{-\hat{k}} - 2\underbrace{\hat{\jmath}\times\hat{\jmath}}_{0}$$

$$= -6\hat{k} - (-\hat{k}) = -5\hat{k}$$

Torque vector

Torque produced by a force is calculated as:





1) Identify force $\vec{F}$
2) Let $\vec{r}$ be a vector from the pivot to $\vec{F}$
3) rearrange $\vec{r}, \vec{F}$ so that their tails coincide
4) the torque produced by $\vec{F}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Warm Up2