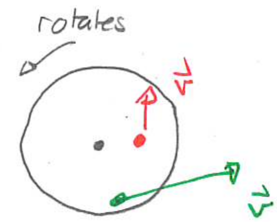
Tues: Warm Up 14

Thurs: Discussion / quiz

Ex: 316, 317, 318, 322, 323, 327

Rotational energy

Consider a uniform disk that rotates about its center. There will clearly be a kinetic energy associated with this, since there is moving mass. How can we account for this?



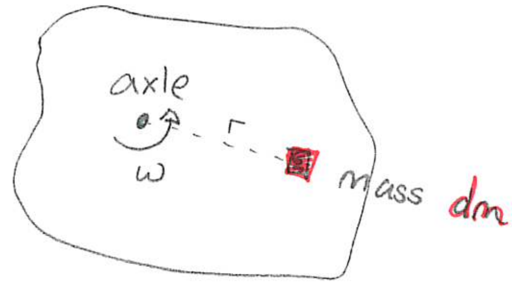
Quiz 1 30% - 80% ; 10% - 20%

With such rotating objects there is a multitude of velocities and the kinetic energy cannot be expressed as some mass $\times$ v^2. However, a general rule is:

> Consider an object that rotates about some axis. This object has rotational kinetic energy
>
> $$K_{rot} = \frac{1}{2} I \omega^2$$
>
> where I = moment of inertia about the axis
>
> ω = angular velocity " " axis

Proof. Consider any flat object rotating about an axle

The shaded piece, with mass dm has kinetic energy



$$dK = \frac{1}{2}\,dm\,v^2$$

where v is its speed. Now

$$v = \omega r.$$

Thus

$$dK = \frac{1}{2}\,dm\,r^2\omega^2 \quad = \quad \frac{1}{2}\,r^2 dm\,\omega^2$$

Adding over all such contributions gives:

$$K = \int dK = \int \frac{1}{2} r^2 dm\,\omega^2$$

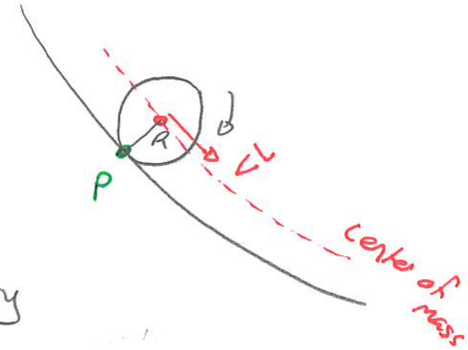
However ω is constant. Thus

$$K = \frac{1}{2}\underbrace{\left[\int r^2 dm\right]}_{\text{moment of inertia } I}\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2} I \omega^2 \qquad \blacksquare$$

Rotational and translational motion

Objects can simultaneously display rotational and translational motion. In this case they will have both translational and rotational kinetic energy. Let P be the point about which the object rotates. Let $\vec{v}$ be the velocity of the center of mass. Then, we identify:



1) the translational kinetic energy (associated with the center-of-mass)

$$K_{trans} = \frac{1}{2} M v^2$$

total mass

speed of c.o.m.

2) the rotational kinetic energy

$$K_{rot} = \frac{1}{2} I \omega^2$$

angular velocity about P

moment of Inertia about c.o.m.

Then the total kinetic energy is

$$\boxed{K = K_{trans} + K_{rot}}$$

Then using Newton's 2^{nd} + 3rd law we can show that

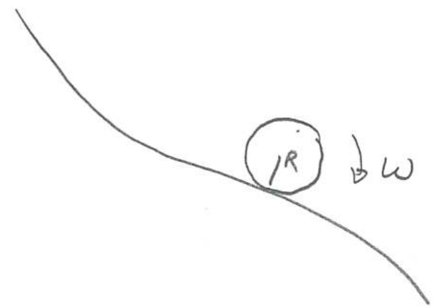
If the net work done by non-conservative forces is zero then the energy

$$E = \underbrace{K}_{K_{trans} + K_{rot}} + U_{grav} + U_{spring} + \dots$$

is constant (conserved)

Rolling without slipping

We consider an object that rolls along a surface without slipping. Suppose the object has a circular cross section with radius R. Then, if the object rolls without slipping, the speed of the center-of-mass satisfies



$$\boxed{V = \omega R}$$

We can incorporate this into the total kinetic energy

$$K = K_{rot} + K_{trans}$$

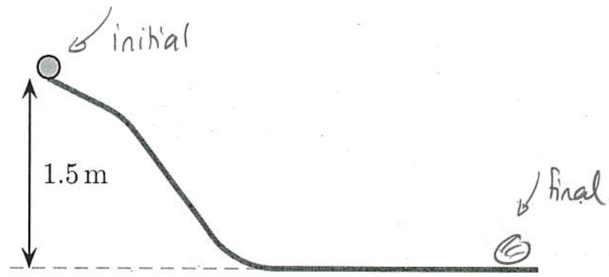
$$= \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

substitute to relate V and ω and remove one of these variables

324 Rolling cylinder

A cylinder with mass M and radius R is released from the top of a track with height 1.5 m. It rolls without slipping. Determine the speed of the cylinder at the bottom of the track. *(131Sp2023)*

rest at



Answer: $E_f = E_i$

$y_i = 1.5m$ $\quad$ $y_f = 0m$

$v_i = 0m/s$ $\quad$ $V_f = ?$

$\omega_i = 0rad/s$ $\quad$ $\omega_f = ?$

$$K_{transf} + K_{rotf} + \cancel{U_{gf}}^{0} = \cancel{K_{trans\,i}}^{0} + \cancel{K_{rot\,i}}^{0} + U_{gi}$$

$$\frac{1}{2} M V_f^2 + \frac{1}{2} I \omega_f^2 = M g y_i$$

For a cylinder $I = \frac{1}{2} M R^2$. Thus

$$\frac{1}{2} \cancel{M} V_f^2 + \frac{1}{2}\frac{1}{2} \cancel{M} R^2 \omega_f^2 = \cancel{M} g y_i$$

Now $V_f = R\omega_f \Rightarrow \omega_f = V_f / R$. Thus

$$\frac{1}{2} V_f^2 + \frac{1}{4} \cancel{R^2} \frac{V_f^2}{\cancel{R^2}} = g y_i$$

$$\Rightarrow \frac{3}{4} V_f^2 = g y_i \quad \Rightarrow \quad V_f^2 = \frac{4 g y_i}{3}$$

$$\Rightarrow V_f = \sqrt{\frac{4 g y_i}{3}} = \sqrt{\frac{4 \times 9.8 m/s^2 \times 1.5m}{3}}$$

$$= 4.4 m/s$$

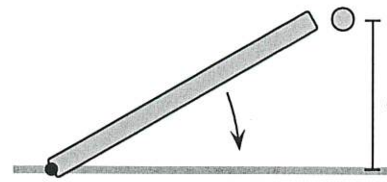
Quiz 2 $\quad$ 30% − 60% $\}$ 30% − 70%

Quiz 3 $\quad$ 20% → 10% $\}$ 6%

Quiz 4

319 Toppling rod versus freely falling ball, 1

A rigid rod with uniformly distributed mass M and length L can pivot about a frictionless axle in a horizontal surface. The rod is held at rest with one end height h above the surface. A ball with mass m is also held at rest alongside the tip of the rod. Both are released at the same time. *(131Sp2023)*



a) Determine the speed of the ball just before it hits the horizontal surface.

b) Determine the speed of the tip of the rod just before it hits the horizontal surface.

c) Which hits first?

Answer a) $E_f = E_i$

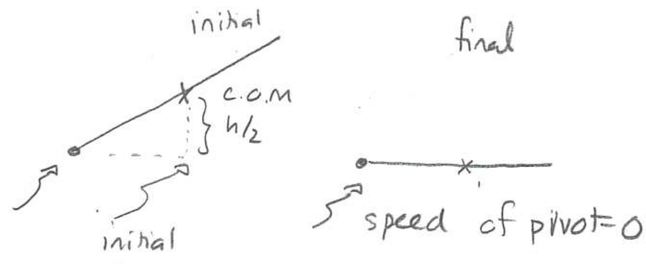
$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2} m v_f^2 + \cancel{mgy_f} = \cancel{\frac{1}{2} m v_i^2} + mgy_i$

$\Rightarrow \frac{1}{2} \cancel{m} v_f^2 = \cancel{m} gh \Rightarrow v_f^2 = 2gh$

$\Rightarrow \boxed{v_f = \sqrt{2gh}}$

b) Use the pivot as an axle

$E_f = E_i$



$K_{rotf} + K_{transf} + \cancel{U_{gf}} = \cancel{K_{roti}} + \cancel{K_{transi}} + U_{gi}$

$\frac{1}{2} I \omega_f^2 + \cancel{\frac{1}{2} M v_f^2} = Mg\frac{h}{2}$

$\Rightarrow \omega_f = \frac{2v_f}{L}$

I about end $= \frac{1}{3} ML^2$

$\frac{1}{2} \frac{1}{3} \cancel{M} L^2 \omega_f^2 = \cancel{M} g\, h/2$

$\Rightarrow \frac{1}{3} L^2 \omega_f^2 = gh$

Now the speed of the tip is $v_f = L\omega_f$

$\Rightarrow \frac{1}{3} v_f^2 = gh \Rightarrow v_f = \sqrt{3gh}$