Phys 131 2023

Lecture 36

Mon: HW by 5pm

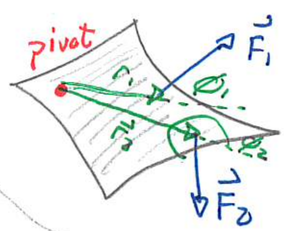
Ex: 291, 293, 296, 298, 299, 301, 308, 309

Tues: Warm Up 14 (D2L)

Rotational dynamics

The conceptual scheme for rotational dynamics is

Specify forces and a pivot



→ Determine moment of inertia about pivot

$$I = \sum m_i r_i^2$$

$$I = \int r^2 dm$$

↓

Determine torque produced by each force (separately for each force)

$$\tau = rF\sin\phi$$

→ Net torque

$$\tau_{net} = \tau_1 + \tau_2 + \ldots$$

$$= \sum_{\text{all forces } i} \tau_i$$

→ Rotational version of Newton's 2nd Law

$$\tau_{net} = I\alpha$$

→ Gives angular acceleration α

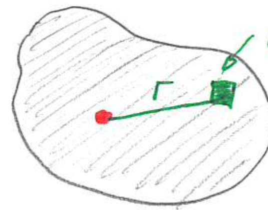
→ Gives rate of change of angular velocity

$$\frac{d\omega}{dt} = \alpha$$

Warm Up 2 (from previous).

Moment of inertia for a continuous mass distribution

In general mass can be continuously distributed in a rotating system and we need a method to determine the moment of inertia in this case. The method will use the rule for point masses



$$I = \sum_{m_i} m_i r_i^2.$$

The strategy is to decomposed the continuous distribution into infinitesimally small pieces. Then each is approximately a point particle. Thus

$$I \approx \sum_{\text{all pieces}} r^2 dm \rightsquigarrow \boxed{I = \int_{\substack{\text{entire}\\ \text{volume}}} r^2 \, dm}$$

helps set up integral

varies throughout volume

These can be calculated for symmetric objects Fig 10.20 pg 502

Examples are:

1) Hoop with radius R, mass M $\Big\}$ $I = MR^2$

2) Solid cylinder radius R, mass M $\Big\}$ $I = \frac{1}{2} MR^2$

Then general theorems apply. For example the parallel axis theorem is:

> Consider two axes. Suppose that one axis passes through the center-of-mass. Suppose that the other axis is parallel and passes through a pivot a distance d from the c.o.m.
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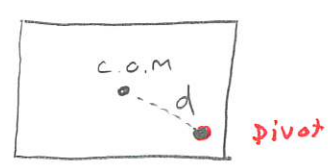
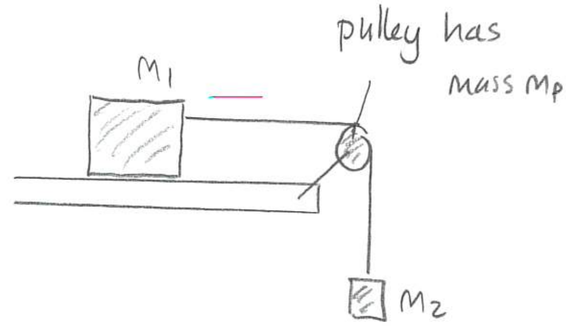
> 
>
>
> Then
>
> $$I_{\text{parallel axis}} = I_{\text{c.o.m.}} + \underbrace{M}_{\text{total mass}} d^2$$

Quiz 1 40% – 90% $\Big\}$ 80%

Rotational motion and connected objects

There are situations that combine rotational and translational motion. If the string does not slip. Then



* suspend mass accelerations

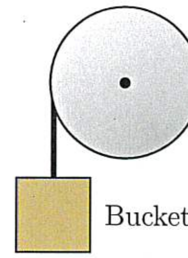
  $\Rightarrow$ string accelerates

  $\Rightarrow$ pulley accelerates

  $\Rightarrow$ net torque on pulley $\neq 0$

Quiz 2 40% – 50% & 70% – 90%

312 Bucket suspended from a rotating pulley



A bucket with mass M is suspended from a massless string that is wrapped around a pulley. The pulley has with radius, R, and uniformly distributed mass, m and rotates about a frictionless axle through its center. The bucket is held at rest 1.5 m above the ground. The aim of this exercise is to find the time taken for the bucket to reach the ground. *(131Sp2023)*

a) Apply Newton's second law to the bucket and determine an expression for the acceleration of the bucket in terms of the tension in the string and other problem variables.

b) Apply the rotational version of Newton's second law to the pulley and use this to determine an expression for the angular acceleration of the pulley in terms of the tension in the string and other problem variables.

c) Relate the angular acceleration of the pulley to the acceleration of the bucket and use this and the previous expressions to find an expression for the acceleration of the bucket in terms of the masses, the pulley radius and g.

d) Determine the time taken for the bucket to reach the ground if its mass is 3.0 kg, the pulley's mass is 2.0 kg and the radius is 0.20 m.

Answer: a) The general strategy is:

Linear dynamics
$\Downarrow$
Linear acceleration of bucket, a

Rotational dynamics
$\Downarrow$
Angular acceleration, α of pulley

No slip of string $\rightsquigarrow$ relate α, a

a)

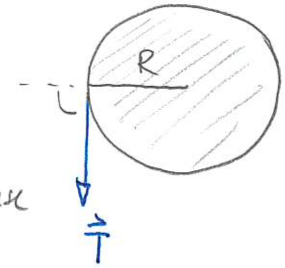
$$\sum F_y = Ma_y \Rightarrow T - Mg = Ma_y.$$

Let a be magnitude of acceleration of bucket

$$\Rightarrow a_y = -a$$

$$\Rightarrow T - Mg = -Ma \Rightarrow \boxed{Ma = Mg - T} \quad -(1)$$

b) $\tau_{net} = I\alpha$



Moment of inertia of pulley

$I = \frac{1}{2}MR^2$

$\tau_{net} = \tau_T + \underbrace{\tau_{axle} + \tau_{grav}}$

both zero because forces are at pivot

$\tau_T = TR\sin 90° = TR$

Thus $TR = \frac{1}{2}MR^2\alpha \Rightarrow T = \frac{1}{2}mR\alpha$ — (2)

c) String does not slip $\Rightarrow$ v string = v rim pulley

$\Rightarrow$ accel " = accel " "

$\Rightarrow a = \alpha R$ — (3)

Combining (3), (2) gives $T = \frac{1}{2}ma$

Then this with (1) $\Rightarrow Ma = Mg - \frac{1}{2}ma$

$\Rightarrow (M + \frac{1}{2}m)a = Mg$

$\Rightarrow a = \left(\frac{M}{M + \frac{1}{2}m}\right)g$

d) Initial

Final

$t_i = 0s$ | $t = ?$
$y_0 = 1.5m$ | $y_f = 0m$
$V_{iy} = 0m/s$ | $V_{fy} = ?$

$\cancel{y} = y_0 + \cancel{V_{0y}t} + \frac{1}{2}a_y t^2$

$\Rightarrow 0 = y_0 + \frac{1}{2}a_y t^2 \Rightarrow -2y_0 = a_y t^2 \Rightarrow t^2 = \frac{-2y_0}{a}$

Then $a = \left(\frac{3.0kg}{3.0kg + \frac{1}{2}2.0kg}\right)g = 0.75g = 7.4m/s^2$

$a_y = -a = -7.4m/s^2$

$t^2 = \frac{-2(1.5m)}{-7.4m/s^2} = 0.405s^2 \Rightarrow t = 0.64s$