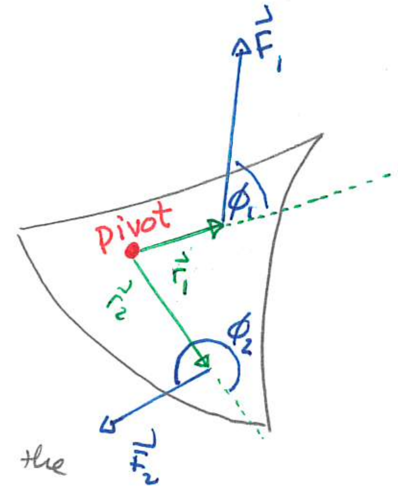
Thurs: Discussion / quiz

Ex 278, 280, 281, 285, 287, 288, 289

Fri: -

Torques and dynamics



The rotational effects of any force can be described using a torque produced by that force. We use

* a moment vector $\vec{r}$ from the pivot to the point of application of the force.
* an angle ϕ measured c.c.w from the moment vector extension to the force.
* then the torque produced is $\boxed{\tau = rF\sin\phi}$

When there are multiple forces present:

* calculate individual torques separately $\tau_1 = r_1 F_1 \sin\phi_1$

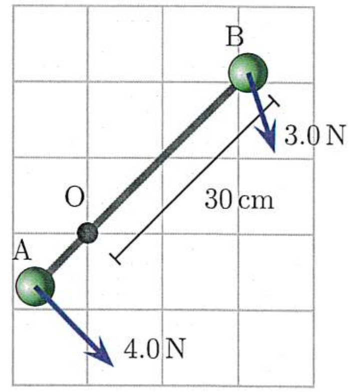
  $\tau_2 = r_2 F_2 \sin\phi_2$

* form the net torque $\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \ldots$

Quiz 1 60% - 90% } 50% - 70%

304 Rotational dynamics of a barbell

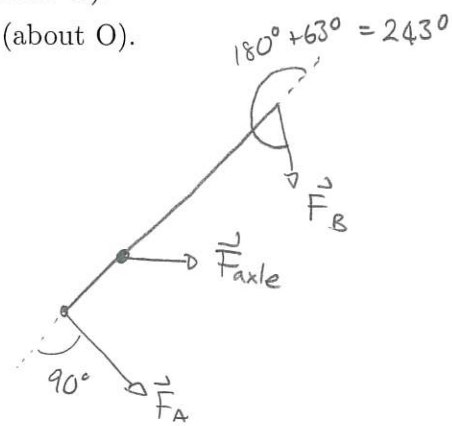
A rigid barbell consists of two heavy balls mounted at the ends of a light rigid 40 cm long rod. The barbell can rotate about an axle (pointing perpendicular to the board/page) at O. The mass of A is 600 g, the mass of B is 300 g and the mass of the rod is negligible. One force acts on each ball and the force on ball A is perpendicular to the rod. The angle between the force on B and the rod is 63°. The set-up is such that gravitational forces are irrelevant. At an indicated moment the rod makes a 45° angle with respect to the usual x axis. *(131Sp2023)*



a) Determine the net torque on the barbell (about O).

b) Determine the moment of inertia of the barbell (about O).

c) Determine the angular acceleration of the barbell (about O).



Answer: a) $\tau_{net} = \tau_{axle} + \tau_A + \tau_B$

Force A: $\tau_A = r_A F_A \sin 90°$

$\tau_A = 0.10\text{m} \times 4.0\text{N} \sin 90°$

$= 0.40\,\text{Nm}$

Force B: $\tau_B = r_B F_B \sin 243°$

$= 0.30\text{m} \times 3.0\text{N} \sin 243°$

$= -0.80\,\text{N·m}$

Force axle: $\tau_{axle} = \underbrace{r_{axle}}_{=0} F_{axle} \sin\phi = 0\,\text{N·m}$

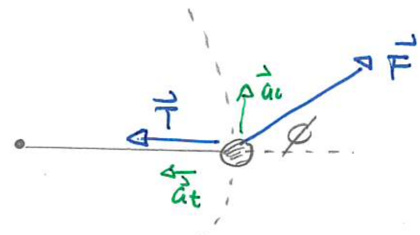
Thus $\tau_{net} = 0.40\,\text{N·m} - 0.80\,\text{N·m} = -0.40\,\text{Nm}$

The actual acceleration will depend on the mass and the distribution of mass of the object

Demo: Rotation sticks.

We can illustrate this via an object swinging in a circle on a horizontal frictionless circle.

According to Newton's 2nd Law



$$\vec{F}_{net} = m\vec{a}$$

$$\Rightarrow \quad \vec{F} + \vec{T} = m\vec{a}$$

But $\vec{a} = -a_c\hat{\imath} + a_t\hat{\jmath}$ gives:

$$\vec{F} + \vec{T} = m(-a_c\hat{\imath} + a_t\hat{\jmath}) \quad \Rightarrow \quad F\cos\phi\,\hat{\imath} + F\sin\phi\,\hat{\jmath} - T\hat{\imath} = -ma_c\hat{\imath} + ma_t\hat{\jmath}$$

Comparing the $\hat{\jmath}$ components gives:

$$F\sin\phi = ma_t = m\alpha r \qquad \Rightarrow \qquad rF\sin\phi = mr^2\alpha$$

This is the torque produced by F. Thus we get, in this case,

$$\tau = mr^2\alpha.$$

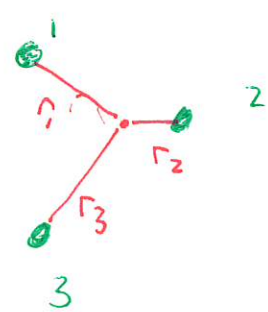
So we define:

The moment of inertia of a system of objects about a pivot is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$$
$$= \sum_{\text{all objects}} m_ir_i^2$$

where m_i = mass of object i

r_i = distance from object i to pivot



Units: kg m²

The same applies for continuous mass distributions where the sum is replaced by the integral.

Warm Up1

The net torque on an object determines an object's angular acceleration via the following rule whenever the axis of rotation is fixed:

$$\boxed{\tau_{net} = I\alpha}$$

where I is the moment of inertia of the object

Quiz2 60% - 90%

Warm Up2

Ex 304 continued

b) $I = \sum m_i r_i^2 = m_A r_A^2 + m_B r_B^2$

$= 0.600\text{kg} \times (0.10\text{m})^2 + 0.300\text{kg} \times (0.30\text{m})^2$

$= 0.033\text{kg m}^2$

c) $\tau_{net} = I\alpha$

$\Rightarrow -0.40\text{Nm} = 0.033\text{kgm}^2\, \alpha$

$\Rightarrow \alpha = \dfrac{-0.40\text{mN}}{0.033\text{kgm}^2} = -12.1\text{rad/s}^2$

Thus the angular velocity changes by -12.1rad/s every second.