Thurs: Review 3

Fri: Exam 3 Covers Ch 7,8,9

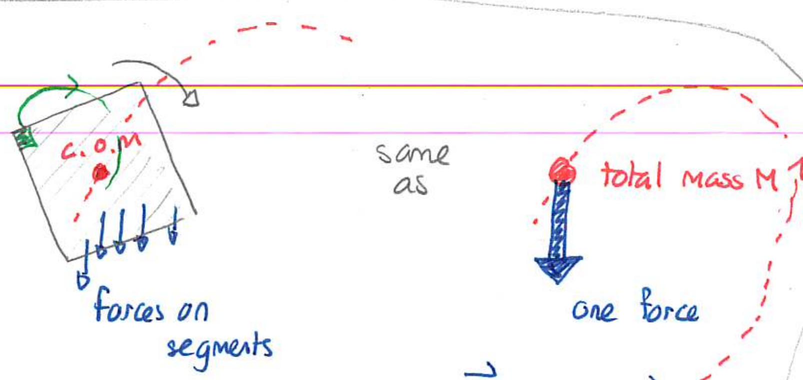
Lectures 23 - 32

HW 8-9

Motion of "bulk" objects

Newton's laws strictly refer to point particles and describe their motion via acceleration. For an extended bulk object the same laws predict that

The center-of-mass moves as though it were a point particle with all the external forces acting on the point particle and all the mass at the point particle

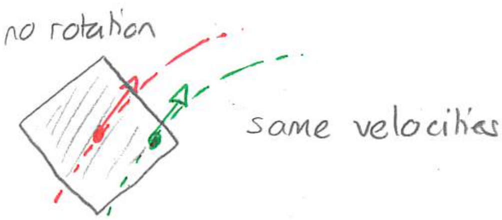


$$\vec{F}_{net} = M\vec{a}$$

acceleration predicts trajectory

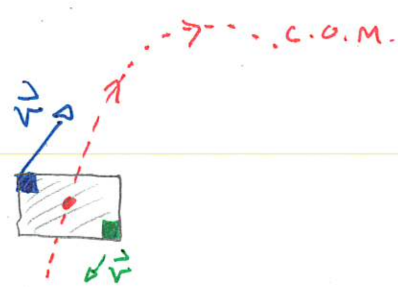
Provided that the object does not rotate then the motion of the center of mass is sufficient to describe the object's motion since at each instant every point has the same velocity.

However when an object rotates, whether the center of mass is at rest or not, different parts move with different velocities



Rotating objects

Consider an object that rotates while the center-of-mass moves. Then different portions have different velocities and speeds. These differences constantly change.

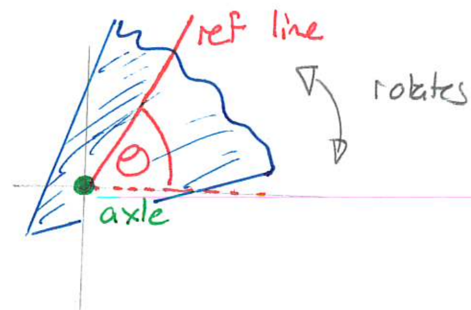


Demo: Throw box with marked corners.

We aim to isolate the rotational component and describe this alone. Rotational motion is widespread in physics and examples include:

1) rotating wheels, disks
2) pivoting beams, rods
3) rotating stars, planets, galaxies
4) atomic + molecular physics

Rotational kinematics



The basic concepts of kinematics and Newtonian dynamics need to be adapted into a suitable form for rotational motion. Consider a rigid object that can rotate about a fixed axle. We could track its configuration via an angle to a reference line on the object. Then:

Angular position $\rightsquigarrow$ θ measured in radians counterclockwise from a reference line.

Angular velocity $\rightsquigarrow$ $\omega = \dfrac{d\theta}{dt}$

units: rad/s

$\rightsquigarrow$ ω = slope of θ vs t

$\rightsquigarrow$ $\omega > 0 \Rightarrow$ rotates counterclockwise
$\omega < 0 \Rightarrow$ rotates clockwise.

Angular acceleration $\rightsquigarrow$ $\alpha = \dfrac{d\omega}{dt}$

Units: rad/s^2

$\rightsquigarrow$ α = slope of ω vs t

Quiz 1 30% - 80% $\}$ 20% - 50%

Quiz 2 60% - 70% $\}$ 50% - 90%

Demo: PhET Ladybug Revolution

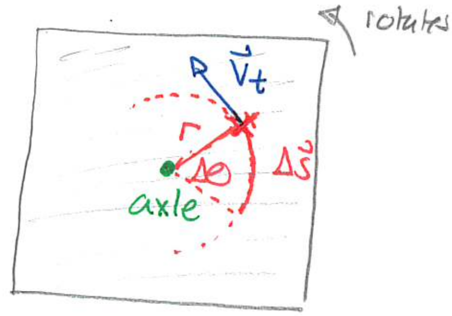
a) $\alpha = 0$ $\quad \omega_i < 0$
b) $\alpha > 0$ $\quad \omega_i < 0$
c) $\alpha < 0$ $\quad \omega_i < 0$

$\}$ show ω vs t

Relationship between linear and angular kinematic quantities

Any point on a rotating object can be described using either angular or linear velocity. These quantities must be related

Then



1) the velocity of the point is a vector tangent to the trajectory. This is the tangential velocity, $\vec{V}_t$

2) the distance traveled in time Δt is denoted Δs and

$$\Delta s = r\Delta\theta$$

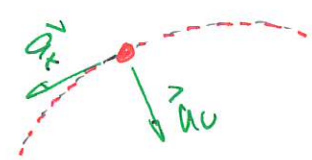
where $\Delta\theta$ is the angular displacement and r is the distance from the axle to the point

3) the speed (magnitude of the tangential velocity) is

$$V_t = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \Rightarrow \boxed{V_t = \omega r}$$

4) the acceleration has two components:

a) radially inward centripetal acceleration $\vec{a}_c$



$$a_c = \frac{V_t^2}{r} = \omega^2 r$$

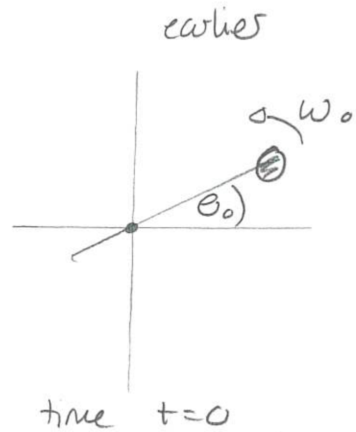
b) a tangential component

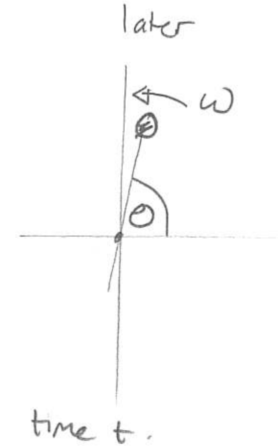
$$a_t = \alpha r$$

Warm Up!

Constant angular acceleration kinematics

If the angular acceleration is constant then integration gives the rotational kinematic equations.





$$\boxed{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \end{aligned}}$$

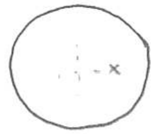
282 Slowing turntable

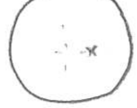
A turntable (circular disk with an axle perpendicular to the disk through its center) initially rotates counterclockwise about the axle at 4800 rpm (revolutions per minute) and subsequently slows at a constant rate. It stops after exactly 30 revolutions. *(131Sp2023)*

a) Determine the angular acceleration of the turntable.

b) Determine the speed of a point 0.15 m from the turntable axle at the instant when it has completed 15 revolutions.

Answer: a) Initial | Final





$\theta_0 = 0\,\text{rad}$ | $\theta = 30 \times 2\pi\,\text{rad}$

$\theta = 60\pi\,\text{rad} = 188\,\text{rad}$

$\omega_0 = 503\,\text{rad/s}$ | $\omega = 0\,\text{rad/s}$

$$\omega_0 = 4800\,\frac{\text{rev}}{1\,\text{min}}$$

$$= 4800\,\frac{\text{rev}}{1\,\text{min}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}$$

$$= 160\pi\,\text{rad/s}$$

$$= 503\,\text{rad/s}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$(0\,\text{rad/s})^2 = (503\,\text{rad/s})^2 + 2\alpha\ 188\,\text{rad}$$

$$\Rightarrow \quad \alpha = -\frac{(503\,\text{rad/s})^2}{376\,\text{rad}} \quad \Rightarrow \quad \alpha = -670\,\text{rad/s}^2$$

b) Here $v_t = \omega r$ where ω = angular velocity after 15 revolutions

↳ $30\pi\,\text{rad} = 94\,\text{rad}$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$= (503\,\text{rad/s})^2 + 2(-670\,\text{rad/s}^2)($$

$$= 1.27 \times 10^5\,(\text{rad/s})^2$$

$$\Rightarrow \omega = 356\,\text{rad/s}$$

$$v_t = \omega r = 356\,\text{rad/s} \times 0.15\,\text{m}$$

$$= 53\,\text{m/s}$$