Tues: Warm Up 12

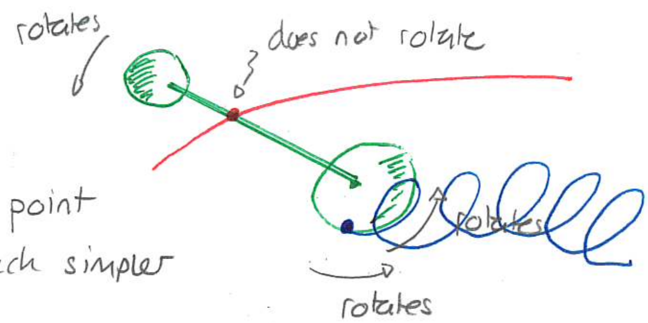
Thurs: Review

Fri: Test 3 Ch 7, 8, 9.

Center-of-mass

Newton's Laws strictly apply to idealized point particles. Many ordinary objects are essentially a large collection of point particles. How do Newton's laws describe their motion? We could consider an object which can rotate as it moves

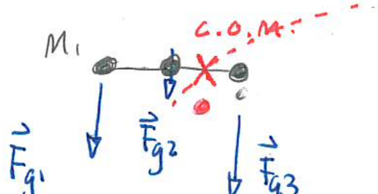
Demo: ~~[illegible]~~
UNSW videos



We see that there is often a single point on the object whose motion is much simpler than that of the remaining points.

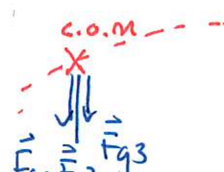
Starting with Newton's Second and Third Laws one can show:

System of objects



External forces acting on objects

Special point center of mass $\rightsquigarrow$ motion is same as point particle with



* mass = total mass of the system, M

* force, $\vec{F}$ is sum all all external forces on the system

sum of all external forces

$$\vec{F} = M\vec{a}$$ ← acceleration of c.o.m

The prescription for finding the center-of-mass is:

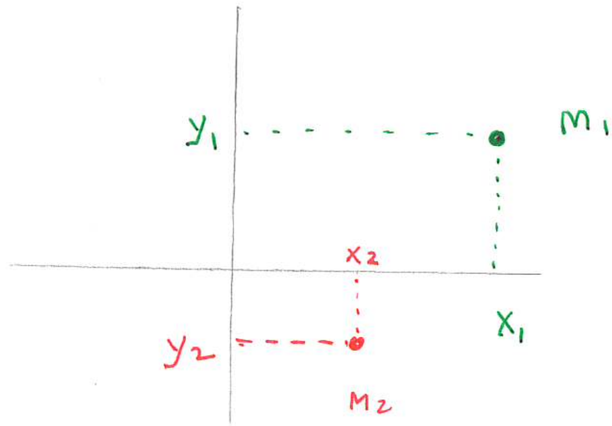
For a collection of point particles, the co-ordinates of the center-of-mass are.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$= \frac{\sum m_j x_j}{\sum m_i}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$= \frac{\sum m_j y_j}{\sum m_i}$$

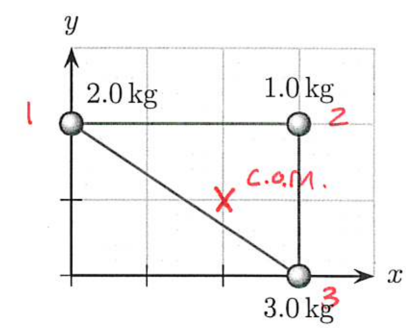


Quiz1 95% & 95%

Note that for collections of point particles, the center-of-mass need not be located on one of the particles. In general it will occupy a location elsewhere.

274 Center of mass of three balls, 1

Three small balls are connected via massless rods in the illustrated configuration. Each grid block is 2.0 cm long. Determine the center of mass of the system. *(131Sp2023)*



Answer: List the data as follows

object	mass	x	y
1	$m_1 = 2.0\,kg$	$0.0\,cm$	$4.0\,cm$
2	$m_2 = 1.0\,kg$	$6.0\,cm$	$4.0\,cm$
3	$m_3 = 3.0\,kg$	$6.0\,cm$	$0.0\,cm$

Then

$$\sum_j m_j = m_1 + m_2 + m_3 = 6.0\,kg$$

and

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_j} = \frac{1}{6.0\,kg}\left[0.0\,cm \times 2.0\,kg + 6.0\,cm \times 1.0\,kg + 6.0\,cm \times 3.0\,kg\right]$$

$$= \frac{1}{6.0\,kg}\,24\,cm\,kg = 4.0\,cm$$

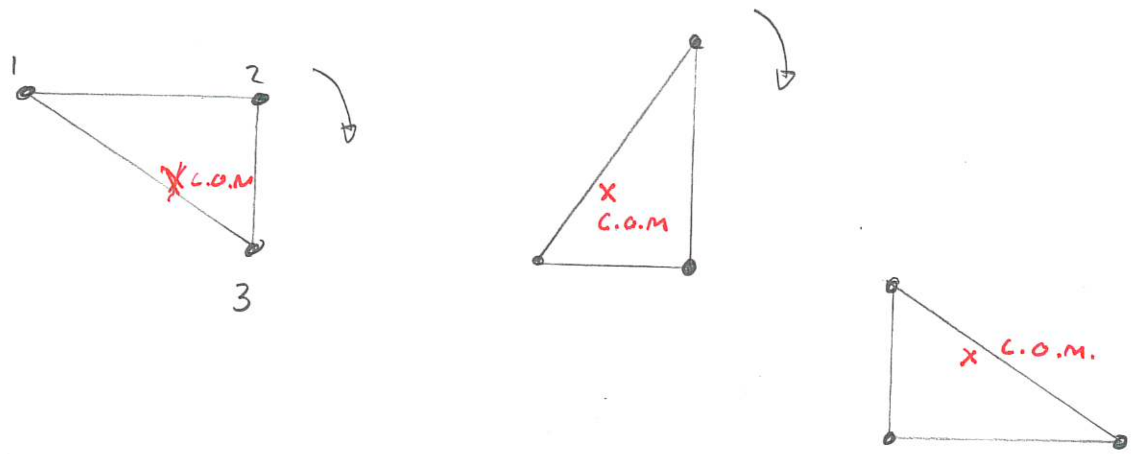
$$y_{cm} = \frac{\sum y_i m_i}{\sum m_j} = \frac{1}{6.0\,kg}\left[4.0\,cm \times 2.0\,kg + 4.0\,cm \times 1.0\,kg + 0.0\,cm \times 3.0\,kg\right]$$

$$= 2.0\,cm$$

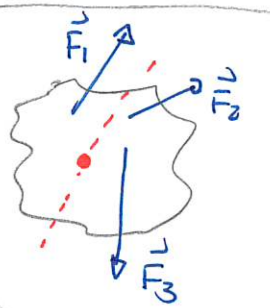
The location is illustrated.

Motion of the center-of-mass

When the objects in the system move the center-of-mass may move. For example if the entire arrangement of the previous exercise rotated the c.o.m would rotate.



The motion of the center-of-mass is determined via



Add all forces on all parts of the system. This is equivalent to adding all external forces

$$\sum_{\text{all forces}} \vec{F}_i = \vec{F}$$

The acceleration of the center of mass $\vec{a}$ is given by

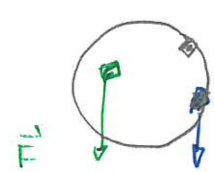
$$\vec{F} = M\vec{a}$$

where M is the total mass of the system

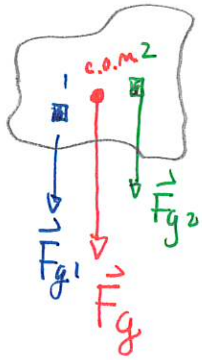
Quiz 2 20% ~ 50% } 40% – 90%

Quiz 3 50% eventually

Demo: Ball thrown → How many forces?



So consider a rigid object. There is a collection of infinitely many point particles. There is a small gravitational force on each. Then



$$\sum \vec{F}_i = \vec{F}_g \quad \text{on entire ball}$$

$$\sum m_i = M \quad \text{of entire ball.}$$

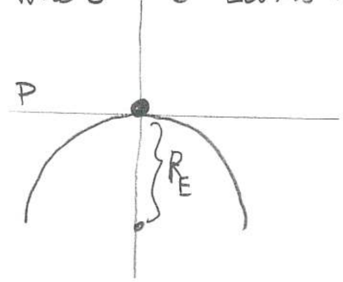
So if the ball is in free fall $\vec{F} = M\vec{a}$ gives the acceleration of the center-of-mass

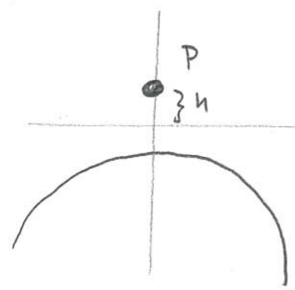
277 Jumping crowd and Earth displacement

A crowd of 100000 people each with mass 80 kg is initially at rest on Earth's surface. They all jump upward simultaneously and their maximum height from their starting point is 0.50 m. Determine the maximum displacement of Earth from its starting point. *(131Sp2023)*

Answer The net external force is zero. The center-of-mass is initially at rest ⇒ it remains at rest.

Consider the people as a single point particle, P and determine where the Earth's radius moves.





Before

$y_{pi} = 0$

$y_{Ei} = -R_E$

$$y_{CM} = \frac{M_p y_{pi} + M_E y_{Ei}}{M_p + M_E}$$

$$y_{CM} = -\frac{M_E}{M_p + M_E} R_E$$

After

$y_{pf} = h$

$y_{Ef} = ??$

$$y_{CM} = \frac{M_p y_{pf} + M_E y_{Ef}}{M_p + M_E}$$

$$y_{CM} = \frac{M_p \cdot h + M_E y_{Ef}}{M_p + M_E}$$

These are equal ⇒ $\dfrac{-M_E R_E}{\cancel{M_p + M_E}} = \dfrac{M_p h + M_E y_{Ef}}{\cancel{M_p + M_E}}$

$\Rightarrow \quad -M_E R_E = M_p h + M_E y_{Ef}$

$\Rightarrow \quad y_{Ef} = \underbrace{-R_E}_{y_{Ei}} \underbrace{- \frac{M_p}{M_E} h}_{\text{shift}}$

The shift is

$$\frac{M_p}{M_E} h = \frac{80\,\text{kg} \times 10^5}{6.0 \times 10^{24}\,\text{kg}} \times 0.50\,\text{m}$$

$$= 6.7 \times 10^{-19}\,\text{m}$$