Phys 131 2023

Lecture 29

Tues: Warm Up 11 (D2L)

Thurs: Discussion / quiz

Ex: 246a, 249, 251ab, 253, 254, 255, 256

note these are different in style to these

Energy exchange

The most general rule that we have for energy is:

The total mechanical energy is $E = K + \underbrace{U_{cons1} + U_{cons2} + \dots}$

potential for each conservative force.

The work done by non-conservative forces is

$W_{nc} = \underbrace{W_{non-cons1} + W_{non-cons2}}_{\text{work for each non-conservative force}} + \dots$

Then between any initial / final point of a system's motion

$$\Delta E = E_f - E_i = W_{nc}$$

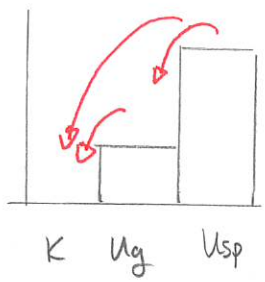
In cases where the non-conservative forces do zero work, $W_{nc} = 0$ and the total energy is conserved

If $W_{nc} = 0 \Rightarrow \Delta E = 0 \Rightarrow \Delta K + \Delta U_{cons1} + \Delta U_{cons2} + \dots = 0$

This allows us to view conservation of energy as an exchange between various types of energy.

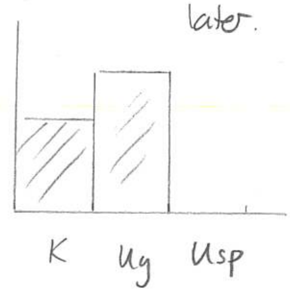
Demo: PhET Springs / Masses

* Energy Tab
* No damping – suspend mass
  – observe energy interchange
* Add damping – thermal energy represents negative work done by damping / air resistance.



earlier

later.



This is analogous to moving money between bank accounts with the different energies representing different bank accounts.

Quiz 1 40% ~ 90% → 30% – 80%

Quiz 2 30% ~ 40% → 60% – 80%

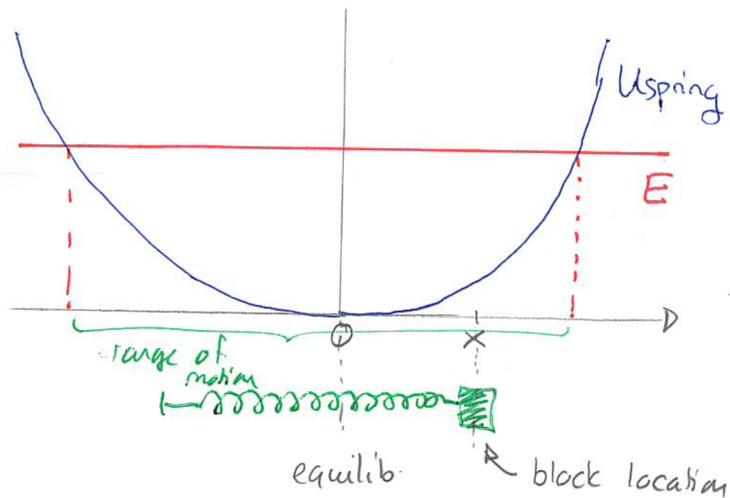
Potential energy graphs and motion

We can use graphical representations of energy to describe qualitative aspects of motion. Typically these consider graphs of potential energy as a function of the co-ordinates that describe the state / configuration of a system.

As an example, consider a spring/mass where the spring moves horizontally. Let $x=0$ be the equilibrium position of the spring. Then.

$$U_{sp} = \frac{1}{2} k x^2$$

and a plot of this yields a parabola. Now suppose that the total energy of the system is E. Then



$$E = K + \cancel{U_g} + U_{sp}$$

0 since at $y=0$

$$E = K + U_{sp} \quad \Rightarrow \quad \boxed{K = E - U_{sp}}$$

But $K = \frac{1}{2} m v^2$ must be positive. Thus

$$\boxed{E > U_{sp}}$$

Place restrictions on locations of object.

Quiz 3 80% } 80% ⇝

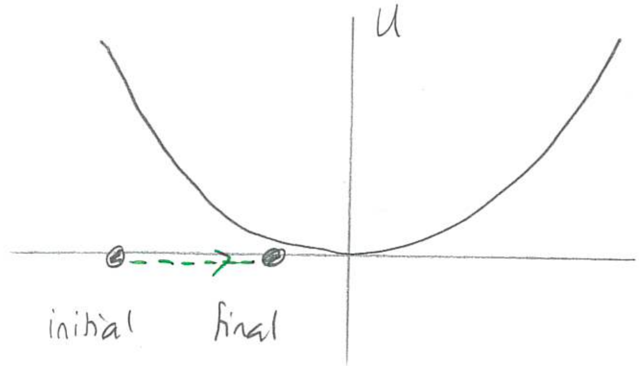
We see that:

1) range of motion ⇝ x such that $U < E$
2) highest speed ⇝ x such that U is lowest
3) turning points ⇝ x such that $U = E$ (briefly stops)
4) equilibrium ⇝ x such that slope $U = 0$

Slide 1

Slide 2 Slide 3

Force and potential energy



Can we find information about forces from a plot of U vs x?

Consider an object moving between two locations as illustrated. Then

U decreases $\Rightarrow$ K increases $\Rightarrow$ v increases.

The object moves right an speeds up. Thus the acceleration and force are to the right. Then:

U has negative slope $\Rightarrow$ force right $\Rightarrow$ $F_x > 0$

U has positive slope $\Rightarrow$ force left $\Rightarrow$ $\underbrace{F_x}_{\text{x component of force}} < 0$

We can apply calculus to the definitions of force, work and potential energy. These give:

> For any object moving along the x-axis under potential $U(x)$ the force associated with the potential is
>
> $$F_x = -\frac{dU}{dx}$$

Thus

$F_x =$ negative slope of U vs x

Quiz 4 90% $\gtrless$ 90%

258 Force for the Lennard-Jones potential

The Lennard-Jones potential for two air molecules is

$$U(x) = \epsilon\left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6}\right]$$

where a is a constant with units of meters and ϵ is a constant with units of energy. *(131Sp2023)*

a) Determine an expression for the force associated with this potential. This is the force that one molecule exerts on the other.

b) Determine the force when $x = a$. For air $a = 3.42 \times 10^{-10}$ m and $\epsilon = 1.65 \times 10^{-21}$ J.

c) Determine the force when $x = 2a$. For air $a = 3.42 \times 10^{-10}$ m and $\epsilon = 1.65 \times 10^{-21}$ J.

Answer: a) $F(x) = -\frac{dU}{dx}$

$$= -\epsilon\left[\frac{d}{dx}\frac{a^{12}}{x^{12}} - \frac{d}{dx}\frac{a^6}{x^6}\right]$$

$$= -\epsilon\left[-12\frac{a^{12}}{x^{13}} - (-6)\frac{a^6}{x^7}\right]$$

$$= 6\epsilon\left[\frac{2a^{12}}{x^{13}} - \frac{a^6}{x^7}\right]$$

b) $F(a) = 6\epsilon\left[\frac{2}{a} - \frac{1}{a}\right] = \frac{6\epsilon}{a} = \frac{6 \times 1.65\times10^{-21}\,\mathrm{J}}{3.42\times10^{-10}\,\mathrm{m}} = 2.89\times10^{-11}\,\mathrm{N}$ $\longrightarrow$

c) $F(a) = 6\epsilon\left[\frac{2a^{12}}{2^{12}a^{13}} - \frac{a^6}{2^7a^7}\right] = \frac{6\epsilon}{a}\left[\frac{1}{2^{12}} - \frac{1}{2^7}\right]$

$$= \frac{6\epsilon}{a}(-0.00757)$$

$$= \frac{6\times1.65\times10^{-21}\,\mathrm{J}}{3.42\times10^{-10}\,\mathrm{m}}(-0.00757)$$

$$= -2.19\times10^{-13}\,\mathrm{N}$$

$\longleftarrow$