Mon: HW 5pm

Ex 227, 229, 232, 237, 238, 240, 242, 245

Tues: Warm Up (D2L)

Energy conservation

So far we have a system of energy conservation for situations involving gravity and springs

If gravity and springs are the only forces that do non-zero work on a system then the mechanical energy

$$E = K + U_{grav} + U_{spring\,1} + U_{spring\,2}$$

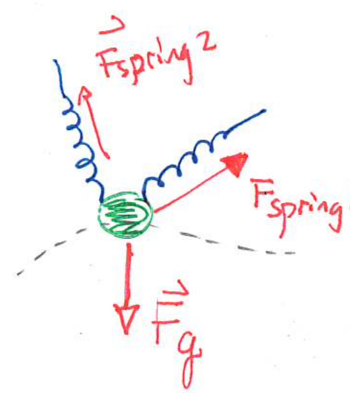
is constant throughout the motion of the system

Here

$$K = \frac{1}{2}mv^2$$

$$U_{grav} = mgy$$

$$U_{spring} = \frac{1}{2}kx^2$$



Quiz 1 70% − 90% ≥ 90%

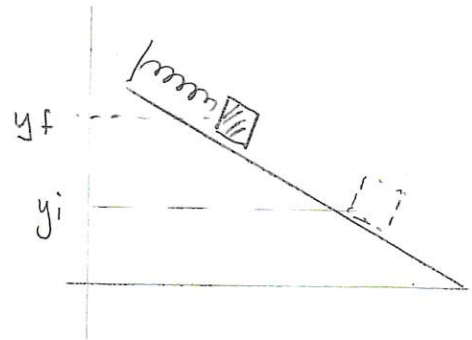
In the previous energy conservation rule, the potential energies completely account for the work done by the associated forces. Is this always possible?

tension force → work done by tension ⇝ some type of tension energy?

friction " → " " " friction ⇝ some type of friction energy?

Conservative forces

Recall that for gravity



$$W_{grav} = -mgy_f + mgy_i$$

$$= -U_{gf} + U_{gi} = -\Delta U_g$$

Similarly for a spring

$$W_{spring} = -\Delta U_{spring}$$

These are two examples of

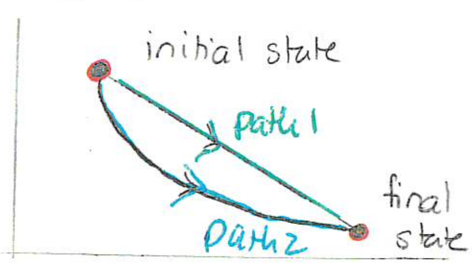
$$W_{force} = -\Delta U_{force}$$

where U_{force} is the potential energy associated with the particular force. This potential energy only depends on the state of the system. For the two forces considered so far

Force	Potential U_{force}	Depends on state via
gravity	$U_g = mgy$	mass of object, vertical position
spring	$U_{spring} = \frac{1}{2}kx^2$	spring constant, spring stretch / compression

these don't depend on how the object moves / changes position.

This has an important implication for the work done. Consider an object starting in an initial state and making its way to a final state via one of two possible paths. Then



$$W_{grav}\,(\text{path } 1) = W_{grav}\,(\text{path } 2)$$

since the work done by gravity only depends on the initial and final states. This can be extended to any paths:

The work done by gravity between an initial location (state) and final location (state) does not depend on the path between these locations.

These are examples of <u>conservative forces.</u>

A force is called conservative. $\Longleftrightarrow$ The work done by the force only depends on the initial and final configuration/state of the system and not the trajectory between these.

$\Updownarrow$

$\Longleftrightarrow$ connection via calculus in three dimensions

There exists a potential energy U_{force}, that only depends on the state of the system such that the work done by the force is

$$W_{force} = -\Delta U_{force}$$

Thus we already have two examples of conservative forces:

1) gravity (near Earth's surface)
2) spring force

Is this true for any force?

Quiz 2 70% – 100% $\rightsquigarrow$ 50% – 70%

Quiz 3 30% $\rightsquigarrow$ 40% – 50%

We see that because work done depends on the path taken:

Friction is non-conservative

Other examples of non-conservative forces are:

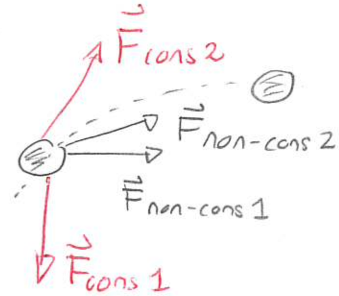
1) tension
2) normal force.

So there is no possible potential energy for:

1) friction
2) tension
3) normal force..

Mechanical energy in general.

Can we still use $E = K + U_{grav} + \ldots$ if there are non-conservative forces that do work? We can by dividing the forces into conservative and non-conservative.



Then

$$\Delta K = W_{net}$$

$$= W_{cons\,1} + W_{cons\,2} + \ldots$$

$$+ W_{non-cons\,1} + W_{non-cons\,2} + \ldots$$

Then for conservative forces there is a potential s.t. $W_{cons\,j} = -\Delta U_{cons\,j}$.

So

$$\Delta K = -\Delta U_{cons\,1} - \Delta U_{cons\,2} - \Delta U_{cons\,3} \ldots + W_{non-cons\,1} + \ldots$$

$$\Rightarrow \Delta K + \Delta U_{cons\,1} + \Delta U_{cons\,2} + \ldots = W_{non-cons\,1} + W_{non-cons\,2} + \ldots$$

So we define the total energy as

$$E = K + U_{cons\,1} + U_{cons\,2} + \ldots$$

and the total work done by all non-conservative forces:

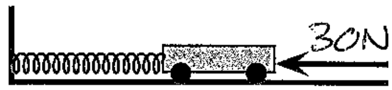
$$W_{nc} = W_{non-cons\,1} + W_{non-cons\,2} + \ldots$$

This gives

$$\boxed{\Delta E = W_{nc}}$$

248 Spring, cart and hand

A 5.0 kg cart can slide along a frictionless horizontal surface. A spring, with spring constant 400 N/m connects the cart to a wall. The spring is initially compressed by 0.25 m and the cart is held at rest. It is released and subsequently a hand pushes with a constant 30 N force against the spring as the spring relaxes. Determine the speed of the cart when the spring reaches its equilibrium position. *(131Sp2023)*



Ans: Initial: spring released

Final: at equilibrium

$v_i = 0\,m/s \qquad v_f = ?$

$x_i = 0.25\,m \qquad x_f = 0$

$y_i = 0 \qquad y_f = 0$

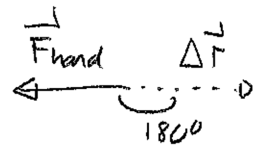
$$\Delta E = W_{nc}$$

where

$$E = K + U_{grav} + U_{spring}$$

$$W_{nc} = W_{hand}$$

Then $W_{hand} = F_{hand}\,\Delta r \cos\theta$



$$= 30N \times 0.25\,m \cos 180^\circ$$

$$= -7.5J$$

Now $\Delta E = W_{nc}$

$\Rightarrow \quad E_f - E_i = W_{hand}$

$\Rightarrow \quad E_f = E_i + W_{hand}$

$$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi} + W_{hand}$$

$$\frac{1}{2}mv_f^2 + \cancel{mgy_f} + \cancel{\frac{1}{2}kx_f^2} = \cancel{\frac{1}{2}mv_i^2} + \cancel{mgy_i} + \frac{1}{2}kx_i^2 - 7.5J$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2 - 7.5J$$

$$\frac{1}{2}\,5.0kg\,v_f^2 = \frac{1}{2}\,400N/m\,(0.25m)^2 - 7.5J$$

$\Rightarrow \quad 2.5kg\,v_f^2 = 12.5J - 7.5J \quad \Rightarrow \quad v_f^2 = \dfrac{5.0J}{2.5kg} = 2\,m^2/s^2 \Rightarrow v_f = 1.4\,m/s$