Tues: Warm Up 10 / Group Exercise

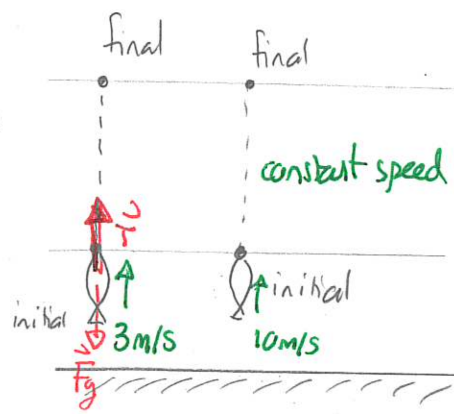
Thurs : Discussion

Ex 201, 204, 205, 217, 218, 221, 223, 224

Thurs: Seminar

Power:

Work and kinetic energy consider the trajectory that an object follows but not the time taken to traverse this trajectory. For example the same object can be raised through the same distance but at different constant speeds. If the speeds are constant then the tension $T = mg$ and the works done will be the same. The change in kinetic energy $\Delta K = 0$ will be the same.



Information about time can be included via power

power ≈ rate at which work is done / energy is delivered

A formal definition is

Let W be the work done by a force along a trajectory. Let Δt be the time elapsed. The power delivered is

$$P = \frac{W}{\Delta t}$$

units: Watts $W = J/s$

If the amount of energy delivered is ΔE then

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

Quiz 1 : 70% – 95% { 40% – 100%

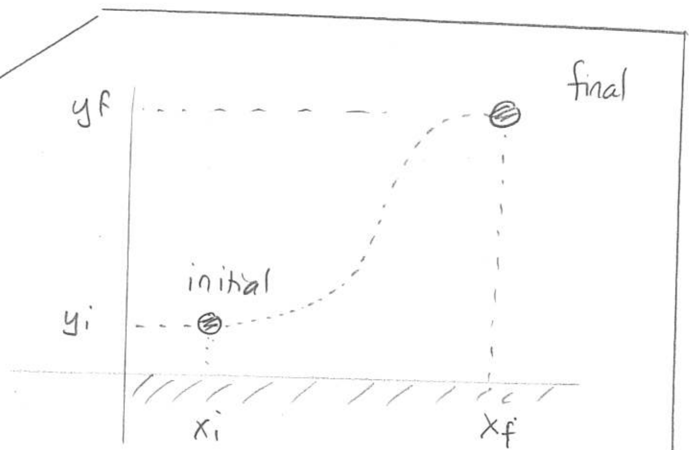
Work done by gravitational forces

Since gravity is ubiquitous in basic mechanics we need strategies for calculating the work done by gravity when an object moves along a general curve trajectory. We will show:

The work done by gravity (near Earth's surface) on an object with mass m is

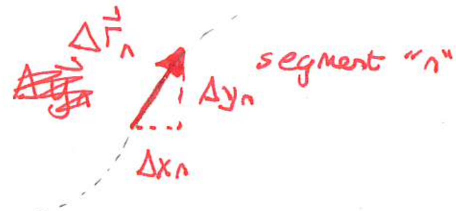
$$W_{grav} = -mg\,\Delta y = -mg(y_f - y_i)$$

where y_i = initial vertical position
y_f = final vertical position



Proof: The curved trajectory can be broken into a succession of small approximately straight trajectories.

Let $\Delta \vec{r}_n$ be the displacement along the n^{th} segment. Then



$$\Delta \vec{r}_n = \Delta x_n \hat{\imath} + \Delta y_n \hat{\jmath}$$

$$\vec{F}_g = -mg\hat{\jmath}$$

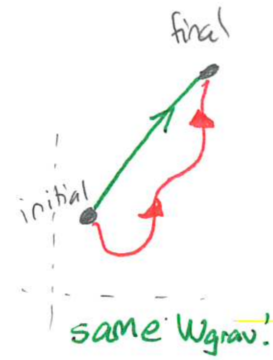
and the work done on this segment is $W_n = \vec{F}_g \cdot \Delta \vec{r}_n = -mg\,\Delta y_n$.

The total work done is

$$W = \sum_n W_n = \sum -mg\,\Delta y_n = -mg \sum_n \Delta y_n = -mg(y_f - y_i)$$

There is an important fact.

> The work done by gravity only depends on the initial and final points of the trajectory



Gravitational potential energy

Consider the special situation where gravity is the only force that does non-zero work. Then the work-kinetic energy theorem gives

$$\Delta K = W_{net} = W_{grav}$$

$$\Rightarrow \quad K_f - K_i = -mg(y_f - y_i)$$

$$\Rightarrow \quad K_f + mgy_f = K_i + mgy_i$$

Thus we define

> The gravitational potential energy of an object with mass m and vertical position y is
>
> $$U_{grav} = mgy$$
>
> Units: J

Thus in the restricted situation under consideration

$$K_f + U_{grav\,f} = K_i + U_{grav\,i}$$

So we define the total mechanical energy of this system

$$\boxed{E = K + U_{grav}}$$

Thus

> If gravity is the only force that does non-zero work on a system then between any initial and final points
>
> $$E_f = E_i \quad \Leftrightarrow \quad K_f + U_{grav\,f} = K_i + U_{grav\,i}$$
>
> and the total mechanical energy is constant along the trajectory

This is an example of the conservation of energy. Alternative statements are

$$E_f = E_i \iff K_f + U_{grav\,f} = K_i + U_{grav\,i}$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$\Delta E = 0 \iff \Delta K + \Delta U_{grav} = 0 \iff \Delta K = -\Delta U_{grav}$$

Quiz 2 30% – 90% } 50% – 60%

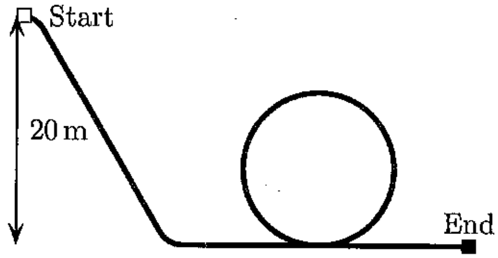
Quiz 3 50% – } 80% –

PhET Energy Skate Park (Basics)

- Intro Tab
- W Track
- Energy Bar Graph -

236 Loop-the-loop rollercoaster

A 100 kg rollercoaster starts from rest at the top of the illustrated track. The rollercoaster completes the loop. Determine speed of the rollercoaster at the end of the track. Ignore friction and air resistance. *(131Sp2023)*



Answer: Initial = start track

Final = end track.

$y_i =$ $\qquad$ $y_f =$

$v_i =$ $\qquad$ $v_f =$

The normal force does zero work.

$$E_f = E_i$$

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_f^2 + \cancel{mgy_f}^{\,0} = \cancel{\frac{1}{2}mv_i^2}^{\,0} + mgy_i$$

$$\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}gy_i$$

$$\frac{1}{2}v_f^2 = gy_i$$

$$v_f^2 = 2gy_i$$

$$v_f = \sqrt{2gy_i}$$

$$= \sqrt{2 \times 9.8\,m/s^2 \times 20\,m}$$

$$= 20\,m/s$$

$$\Rightarrow \frac{1}{2} 100\,kg\ v_f^2 = 100\,kg \times 9.8\,m/s^2 \times 20\,m$$

$$50\,kg\ v_f^2 = 1.96 \times 10^4\,J$$

$$v_f^2 = \frac{1.96 \times 10^4\,J}{50\,kg}$$

$$v_f^2 = 392\,m^2/s^2$$

$$v_f = \sqrt{392\,m^2/s^2}$$

$$= 20\,m/s$$