Thurs: Review Exam II

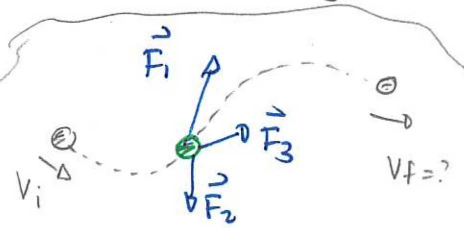
Fri: Exam II

## Work and kinetic energy

Work and kinetic energy offer an alternative to forces and accelerations for assessing motion. The work - kinetic energy theorem states:

> Consider an object following a trajectory from an initial to a final instant. Let
>
> $W_{net}$ = net work done by all forces on object
>
> Then the kinetic energy satisfies:
>
> $$\Delta K = K_f - K_i = W_{net}$$

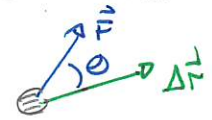
A typical way to use this is




* known initial speed $v_i$ and trajectory
* known forces
* want final speed $v_f$

Get work done by each force

$$W = \vec{F} \cdot \Delta\vec{r}$$
$$= F \Delta r \cos\theta$$




Get net work $W_{net} = W_1 + W_2 + \ldots$

Get initial KE

$$K_i = \tfrac{1}{2} m v_i^2$$

$K_f = K_i + W_{net}$ gives final KE

Get final speed from

$$K_f = \tfrac{1}{2} m v_f^2$$

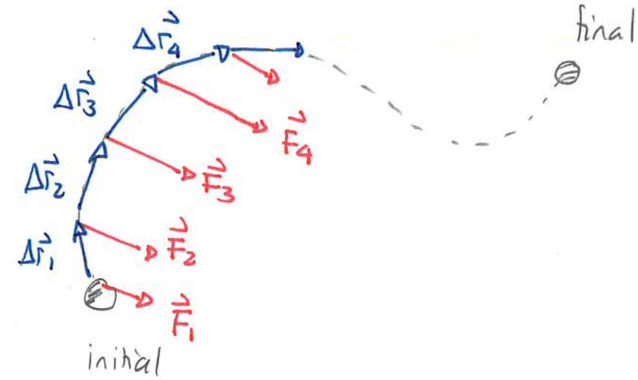
## Work done by a general force

We need to expand the definition of work to account for situations where:

* the force varies during the motion
* the trajectory is not straight.

The strategy is:

* break the trajectory into small segments.
* each segment is approximately straight. The displacement of the $j^{th}$ segment is $\Delta\vec{r}_j$




* the force on each segment is approximately constat. Denote the force on the $j^{th}$ segment $\vec{F}_j$.
* the work done on the $j^{th}$ segment is $\vec{F}_j \cdot \Delta\vec{r}_j$
* the total work done is the sum over all segments

$$W \approx \vec{F}_1 \cdot \Delta\vec{r}_1 + \vec{F}_2 \cdot \Delta\vec{r}_2 + \dots + \vec{F}_j \cdot \Delta\vec{r}_j + \dots = \sum_j \vec{F}_j \cdot \Delta\vec{r}_j$$

This give an approximation that becomes increasingly accurate as the same trajectory is subdivided into increasing numbers of ever smaller segments. This ultimately translates into a (line) integral in multivariate calculus. The work - kinetic energy theorem applies to such line integrals

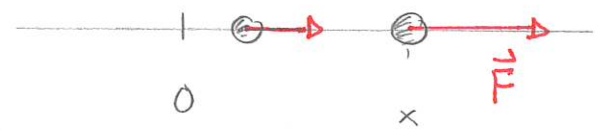
Warm Up 1

~~Quiz 1 80% ~90%~~ $\}$ 90%

## Work done by a variable force for one dimensional motion

A simpler case of this situation is that where the object only moves along one axis and the force is also directed along this axis. For example, suppose that the object moves along the x-axis and that the force acting on the object has form

$$\vec{F} = ax^2\,\hat{\imath}$$




where $a > 0$ is a constant.

Then the work done by this force as the object moves from $x_i$ to $x_f$ is

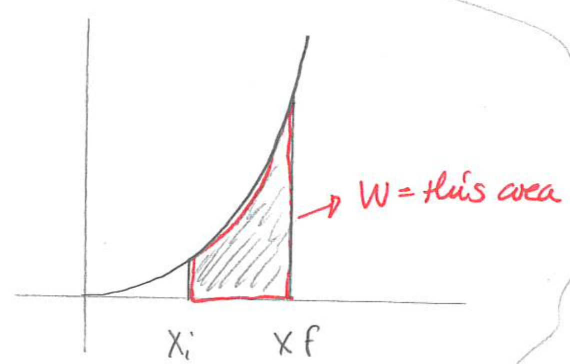
$$W \approx \sum_{\text{all segments } j} F(x_j)\,\Delta x_j$$

and as $\Delta x_j \to 0$ and the number of segments increases.

$$\boxed{W = \int_{x_i}^{x_f} F(x)\,dx}$$

Graphically this means that

Work = area between graph of $F(x)$ and x-axis from $x_i$ to $x_f$




## Spring forces

Springs exert forces that ultimately vary with position. Examples of springs and spring-like forces are

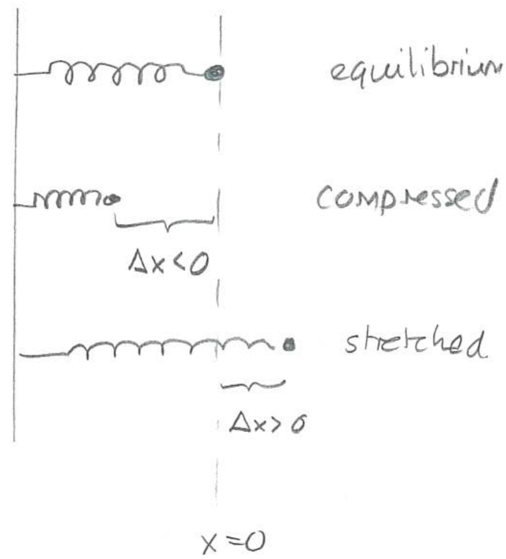
Demo - Springs + masses variable springs.

Demo - Chem Tube 3D vibrations

Demo - PhET Normal modes

Observations yield the following force rule for springs:

1) any spring has an equilibrium position where it exerts zero force
2) any spring exerts a restoring force (toward equilibrium) when stretched or compressed.
3) the magnitude of the spring force partly depends on the stiffness of the spring. This is encoded via a spring constant, $k$, which is particular to the spring




The magnitude of the spring force is

$$F = k\,\Delta x$$

Hooke's Law

where $\Delta x$ is the distance the spring is stretched or compressed from equilibrium and $k$ the spring constant (units N/m)

Aligning the $x$ axis along the spring with $x=0$ at the equilibrium point we get that the $x$-component of the force is

$$F = -kx$$

Then with this set up:

> The work done by a spring with initial displacement $x_A$ from equilibrium and final displacement $x_b$ is
>
> $$W = \tfrac{1}{2}k{x_A}^2 - \tfrac{1}{2}k\,{x_B}^2$$

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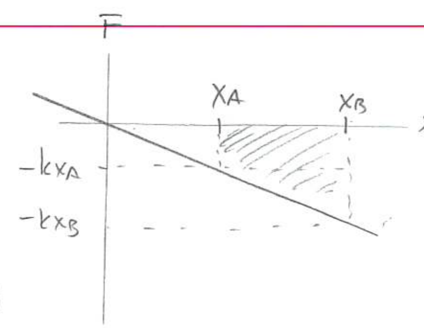
Proof: Consider a graph of $F$ vs $x$




$$W = \text{area under graph} = -kx_A(x_B - x_A) - \tfrac{1}{2}(x_B - x_A)k(x_B - x_A)$$

$$= -k(x_B - x_A)\left[x_A + \tfrac{1}{2}(x_B - x_A)\right] = -k(x_B - x_A)\tfrac{1}{2}(x_B + x_A)$$

$$= -\tfrac{1}{2}k\left({x_B}^2 - {x_A}^2\right) = \tfrac{1}{2}k{x_A}^2 - \tfrac{1}{2}k{x_B}^2 \quad \blacksquare$$

Warm Up 2

## 225 Oscillating block

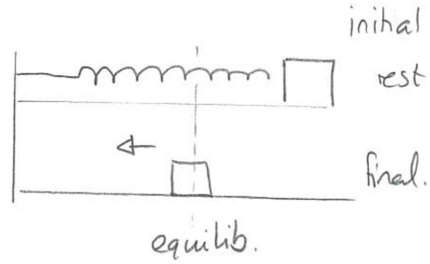
A 6.0 kg block can slide along a frictionless table. It is attached to a spring with spring constant 30 N/m. The block is pulled 0.20 m from the spring's equilibrium position. The block oscillates. *(131Sp2023)*

a) Describe whether you can use constant acceleration kinematics to predict the block's speed as it passes the equilibrium point.

b) Determine the speed of the box as it passes the equilibrium point.

Answer: a) No. The force exerted by the spring $F = -kx$ varies with position and thus the acceleration will not be constant.

b) We can use




$$\Delta K = W_{net}$$

$$= \underbrace{\cancel{W_{normal}}}_{0} + \underbrace{\cancel{W_{grav}}}_{0} + W_{spring}$$

perpendicular to motion

$X_i = 0.20m$ $\quad X_f = 0m$

$V_i = 0m/s$ $\quad V_f = ?$

$$\frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} = \frac{1}{2}kx_i^2 - \cancel{\frac{1}{2}kx_f^2}$$

$$\cancel{\frac{1}{2}}mv_f^2 = \cancel{\frac{1}{2}}kx_i^2$$

$$V_f^2 = \frac{k}{m}x_i^2$$

$$V_f = \sqrt{\frac{k}{m}}\, x_i$$

$$= \sqrt{\frac{30N/m}{6.0kg}}\; 0.20m = \sqrt{5.0s^{-2}}\; 0.20m$$

$$= 0.45m/s.$$