Tues: Warm Up 9 (D2L)

Thurs: Review

Fri: Exam 2

Covers Dynamics / Newton's 2^{nd} Law

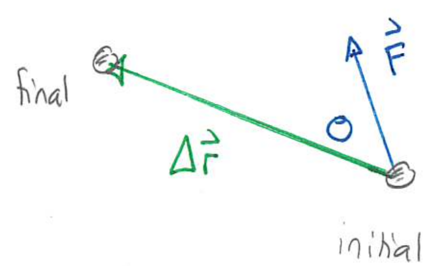
* Ch 5, 6
* Lectures 12 - 22
* HW 5-7

Work done by a constant force

Work is a mathematical quantity that roughly describes the effects of any force on the motion of an object. A preliminary special case is:

Consider an object that moves in a straight line. Suppose that a constant force acts on this object.

Then if $\Delta\vec{r}$ is the displacement of the object and $\vec{F}$ is the force, the work done by the force on the object is



$$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$$

where $F > 0$ is the magnitude of the force
$\Delta r > 0$ is the distance traveled
θ is the angle between $\vec{F}$ and $\Delta\vec{r}$

Quiz 1 20% – 60% ≤ 30% - 80%

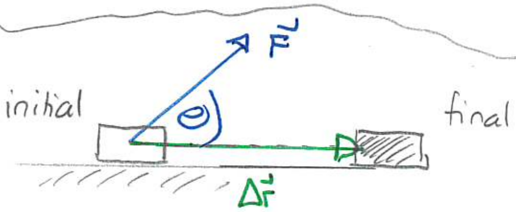
Quiz 2 90%

Quiz 3 30% – 50% ≷ 50%

We need to relate the (abstract mathematical) quantity of work to the motion of an object. We will see that

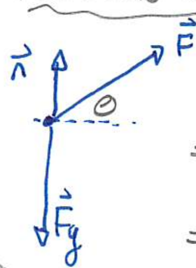
Total work done by all forces on an object → Describes exactly how the object speeds up or slows down.

To illustrate this consider an object pulled along a frictionless horizontal surface



initial speed v_i final speed v_f

→ **Newton's Second Law**



$$\sum F_x = ma_x$$

$$\Rightarrow F\cos\theta = ma_x$$

$$\Rightarrow a_x = \frac{F}{m}\cos\theta$$

Work done by force

Quiz 4 ~70% ≷ 80%

$$W = F\Delta x\cos\theta$$

→ **Constant acceleration kinematics**

$$v_f^2 = v_i^2 + 2a_x\Delta x$$

$$v_f^2 = v_i^2 + 2\frac{F}{m}\cos\theta\,\Delta x$$

$$\Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + F\Delta x\cos\theta$$

→ Combine

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$

This motivates the definition:

> The kinetic energy of an object with mass m, moving with speed v is $K = \frac{1}{2}mv^2$ Units: $kg\,m^2/s^2 = kg\,m/s^2 \cdot m = N\cdot m = J$

We see that the work done by the force in the previous example is related to the change in kinetic energy. This is a special example of a general rule. This general rule requires:

> Suppose that multiple forces act on an object. Then the net work done on the object is
>
> $$W_{net} = W_{force1} + W_{force2} + \dots$$
>
> where the sum includes the work done by each individual force.
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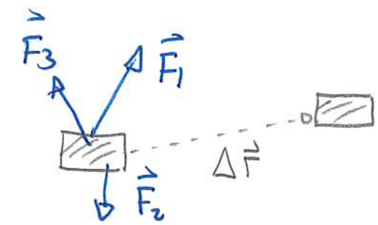
> 
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Quiz 5

So in the example $W_{net} = W_F + \cancel{W_{grav}} + \cancel{W_n} = W_F = \Delta K$. This is an example of the <u>work-kinetic energy theorem</u>

> The kinetic energy of a object changes from an initial to final moment according to:
>
> $$\Delta K = K_f - K_i$$
> $$= W_{net}$$
>
> where W_{net} is the net work done by all forces between the initial and final moments.
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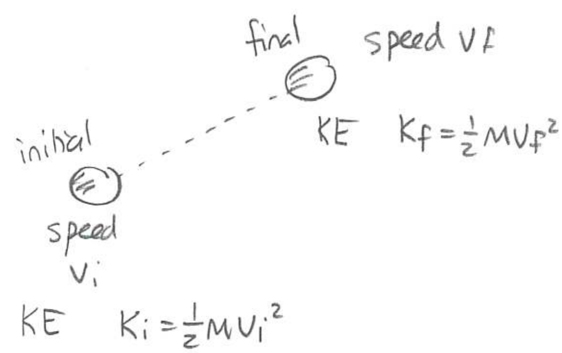
> 
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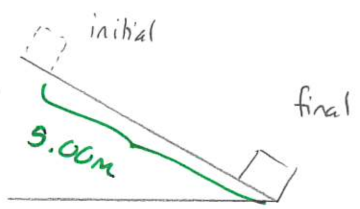
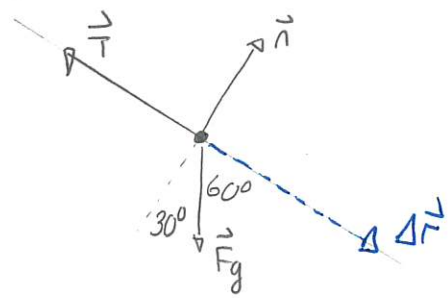
216 Crate lowered down a frictionless ramp

A 20.0 kg crate is at rest at the top of a frictionless ramp that is inclined at 30° from the horizontal. The length of the ramp is 5.00 m. A rope pulls on the crate with a 70.0 N force parallel to and up the ramp. Use work and energy to determine the speed of the crate when it reaches the bottom of the ramp. *(131Sp2023)*

Answer:





$$\Delta K = W_{net}$$
$$= W_T + W_n + W_{grav}$$
$$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_T + W_n + W_{grav}$$

Strategy: * calculate individual works
* substitute + solve for v_f

Individual works

normal: $W_n = n\,\Delta r \cos 90° = 0$

tension: $W_T = T\,\Delta r \cos 180° = -70.0\,N \times 5.00\,m = -350\,J$

gravity: $W_g = F_g\,\Delta r \cos 60° = mg\,\Delta r \cos 60°$

$$= 20.0\,kg \times 9.80\,m/s^2 \times 5.00\,m \cos 60°$$

$$W_g = 490\,J$$

$$\frac{1}{2}mv_f^2 = W_T + W_n + W_{grav}$$
$$\Rightarrow \frac{1}{2}20.0\,kg\,v_f^2 = 490\,J + 0\,J - 350\,J$$
$$\Rightarrow 10.0\,kg\,v_f^2 = 140\,J$$
$$\Rightarrow v_f^2 = 14\,m^2/s^2 \quad \Rightarrow v_f = \sqrt{14.0\,m^2/s^2} \Rightarrow v_f = 3.75\,m/s$$

This illustrates qualitative features of work

Force	work	effect on motion
gravity	positive	speed object up
tension	negative	slow object down
normal	zero	no effect on speed constrains object to ramp.

Then this calculation avoids

* vectors /components
* acceleration
* kinematics