Phys 131 2023

Lecture 23

Mon, March 27 HW by 5pm

Ex: 180, 184, 188, 190, 191, 196, 199, 200

Tues, March 28 Warm Up 9 (D2L)

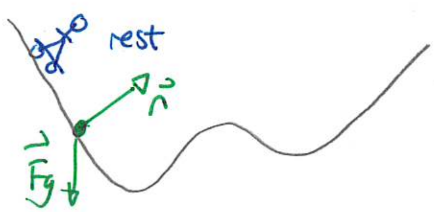
Energy in physics

Newton's system of mechanics allows one to assess, in principle, the motion of any object. In many circumstances the resulting mathematics may be too complicated to admit known solutions

Demo: PhET Energy Skate Park (Basics)

→ Intro → W Shape Track.

→ Release skater.



How long to lowest point?
Speed at lowest point?

Use forces and Newton's Second Law

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$

↳ Predict acceleration

Known initial position

Later positions?

- Forces vary + depend on location.
- Need ~~____~~ locations to describe forces.
- Need forces to describe how location changes

Acceleration depends on location!

??

We will see that in such situations there is an alternative analysis using the concept of energy that will allow for easy answers to some questions.

Demo: PhET Skater

– Energy bar graphs.

In such situations the total energy remains constant and this will ultimately help with the analysis. In classical physics we will develop this via

Newton's Laws + kinematics → Definitions of energy → Conservation of energy

Additional concept: Work.

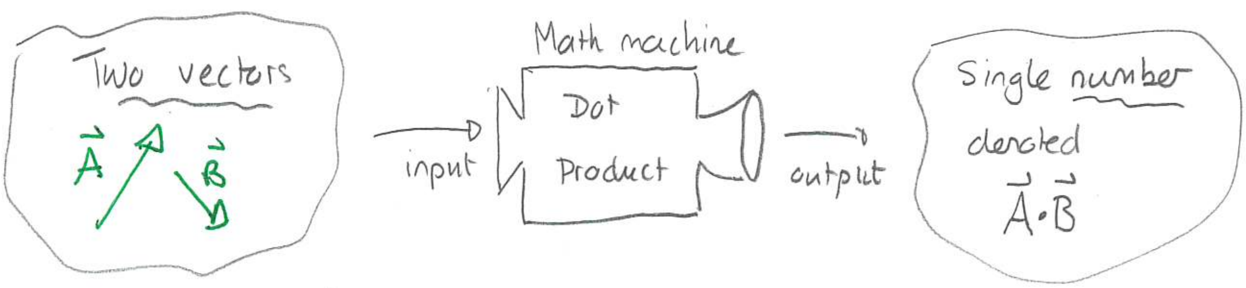
USEFUL

Energy is widespread in physics:

* thermodynamics
* quantum theory
* relativity
* chemistry
* biology
* climate science.

Dot product of vectors Ch 2.4

The concept of work, that is required to develope energy, uses a type of multiplication of two vectors called the dot product.



One definition of the dot product is

Let

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

Then the dot product is

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Quiz 1 60% → 90% 80% — 95%

Quiz 2 80% — 90% 60% → 90%

Quiz 3 95% 95%

Notes:

1) $\vec{A}\cdot\vec{B}$ is a scalar, no unit vectors remain
2) $\vec{A}\cdot\vec{B}$ can be positive, negative or zero

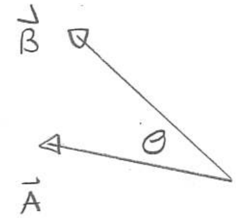
The definition can be used to prove:

1) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
2) $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$
3) $\vec{A}\cdot(\alpha\vec{B}) = \alpha(\vec{A}\cdot\vec{B})$ for any number α
4) $\hat{i}\cdot\hat{i}=1$ $\hat{i}\cdot\hat{j}=0$
   $\hat{j}\cdot\hat{j}=1$ $\hat{j}\cdot\hat{k}=0$
   $\hat{k}\cdot\hat{k}=1$ $\hat{i}\cdot\hat{k}=0$

We can also prove:

For any two vectors

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$



where

$A=$ magnitude of $\vec{A}$
$B=$ " " $\vec{B}$
$\theta=$ angle from $\vec{A}$ to $\vec{B}$

Quiz 4 70% → ?

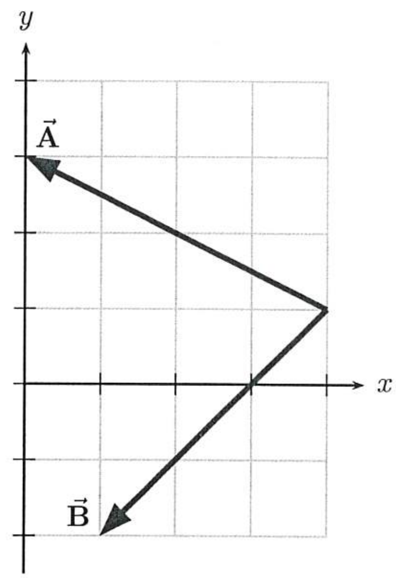
203 Dot product of two vectors, graphical, 2

Consider the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. *(131Sp2023)*

a) Determine the magnitude of each vector.

b) Determine the dot product of the vectors.

c) Use the dot product to determine the angle between the vectors.



Answer:

$$\vec{A} = -4\hat{i} + 2\hat{j}$$

$$\vec{B} = -3\hat{i} - 3\hat{j}$$

a) $A = \sqrt{A_x^2 + A_y^2} = \sqrt{4^2+2^2} = \sqrt{20} = 4.5$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{3^2+3^2} = \sqrt{18} = 4.2$

b) $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$

$= (-4)(-3) + (2)(-3) = 12 - 6 \Rightarrow \vec{A}\cdot\vec{B} = 6$

c) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$6 = \sqrt{20}\sqrt{18}\cos\theta \Rightarrow \cos\theta = \dfrac{6}{\sqrt{20}\sqrt{18}} = \dfrac{6}{\sqrt{4\cdot 5}\sqrt{9\cdot 2}} = \dfrac{1}{\sqrt{10}}$

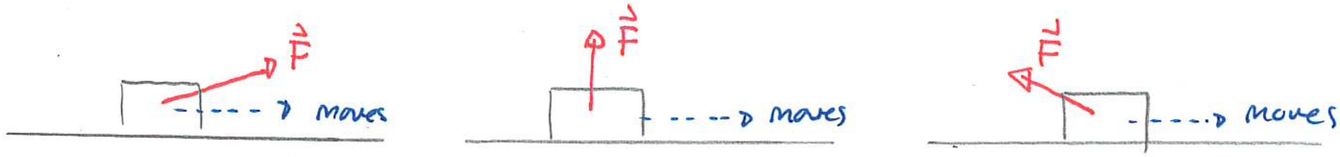
$= 0.316$

$\Rightarrow \theta = \cos^{-1}(0.316)$

$\Rightarrow \theta = 72^\circ$

Work done by a force

Any force acting on an object tends to produce acceleration and thus change the objects velocity. Consider objects moving in a straight line and focus on one of the forces acting on the object.



force causes object to speed up

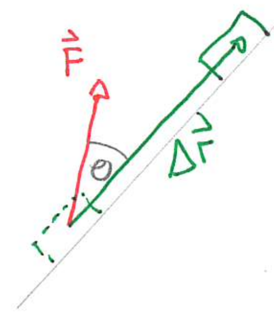
force does not affect speed

force causes object to slow down.

We can account for these effects via the work done by the force.

Suppose that a constant force, $\vec{F}$, acts on an object that is displaced along a straight line, $\Delta\vec{r}$.
Then the work done by the force on the object is

$$W = \vec{F}\cdot\Delta\vec{r}$$



Units: Joules J=N·m

We also get, using the properties of dot product:

$$W = F\,\Delta r\cos\theta$$

where F = magnitude of force >0

Δr = displacement >0

θ = angle between F and Δr

Quiz 5