Phys 131 2023

Lecture 21

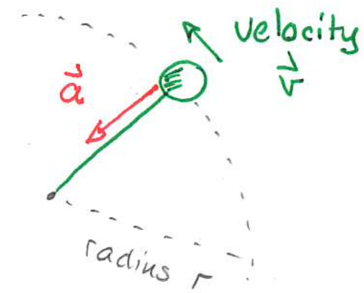
Tues: Warm Up (D2L)

Thurs: & Discussion/quiz

Ex: 181, 182, 183, 186, 187, 189

## Circular Motion

Newton's Laws can be applied to describe the dynamics of circular motion. We first consider uniform circular motion, where the object moves in a circle with constant speed.




What is tension in string?

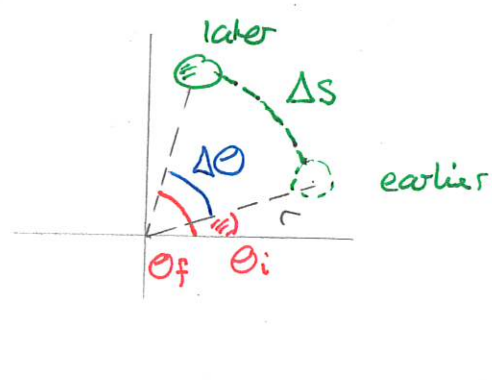
> For uniform circular motion the acceleration of the object is radially inward with magnitude
>
> $$a_c = \frac{v^2}{r}$$
>
> where $v =$ speed
> $r =$ radius of orbit


what gravitational force does Earth exert on Moon?

## Angular velocity

It is convenient to describe circular motion in terms of angular variables rather than Cartesian co-ordinates. We measure angles and angular displacements using radians. This is a measure of angle such that

$$2\pi \text{ radians} = 360°$$



Then the angular displacement of an object moving in a circle is

$$\Delta\theta = \theta_{\underbrace{f}_{\text{later angle}}} - \theta_{\underbrace{i}_{\text{earlier angle}}}$$

and this is related to the distance traveled (arc length) by

$$\Delta s = r\,\Delta\theta$$

It will be convenient to describe the rate of circular motion via

Concept — Angular velocity ~ rate at which angular position changes

and

Mathematical definition. — The angular velocity of an object is

$$\underbrace{\omega}_{\text{omega}} := \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

units : rad/s

Then we get

$$\text{speed } v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{r\,\Delta\theta}{\Delta t} = r \lim \frac{\Delta\theta}{\Delta t} = r\omega$$

Thus

Speed (magnitude of velocity) for an object moving in a circle is

$$v = \omega r$$

Algebraic substitution gives

For an object undergoing uniform circular motion, the centripetal acceleration satisfies

$$a_c = \omega^2 r$$

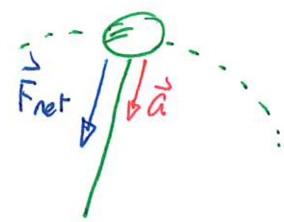
Quiz 1 10% – 100% } 40% – 70%

Quiz 2 80% – 95% } 60% – 95%

Quiz 3 70% – 90% } 50% – 90%

## Dynamics of uniform circular motion

An object in uniform circular motion has a non-zero acceleration. Thus there is a non-zero net force on the object. This is directed radially inward. Thus there must be one or more forces on the object and these add to produce a net inward force.



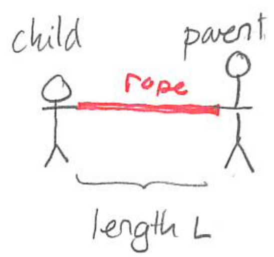

This applies to:

1) objects swinging in circles
2) motion of celestial objects.
3) amusement park rides (dips, loops...)
4) motion of charged particles in magnetic fields.

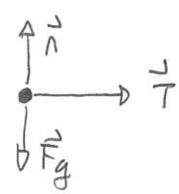
## 185 Child swinging on ice

A parent and child are each on a horizontal sheet of ice. The parent is fixed to the ice and swings the child, who is connected by a horizontal rope to the parent. The child slides without any friction with constant velocity. Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the angular velocity of the child. *(131Sp2023)*

Answer: Horizontal side view




Child

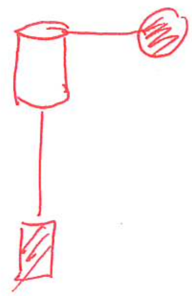



$$\sum F_x = m a_x \Rightarrow T = m a_x \Rightarrow T = m a_c \Rightarrow \boxed{T = m\omega^2 L}$$

$$\sum F_y = m a_y = 0 \Rightarrow n - mg = 0 \Rightarrow n = mg$$

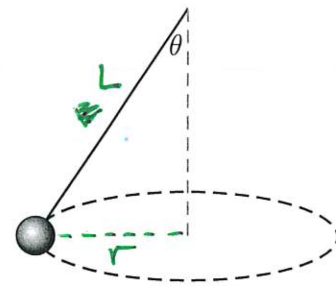
Quiz 4 50% → 70%, & 30% → 50%

Demo:



## 198 Conical pendulum

A ball with mass $m$ swings with a constant speed at the end of a string with length $L$. The angle between the string and the vertical, $\theta$, is constant. Suppose that one can easily measure $L, m$ and $\theta$. We would like to determine expressions for other quantities in terms of these. *(131Sp2023)*

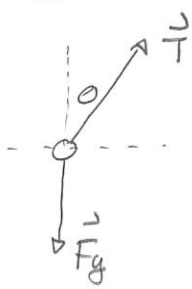



a) Determine an expression for the speed of the ball in terms of $m, \theta, L$ and constants. For a given string length and angle are there multiple possible speeds at which the ball can move? Are there multiple possible masses such that the ball can trace the same path for a given string length and angle?

b) Determine an expression for the angular velocity of the ball in terms of $m, \theta, L$ and constants.

c) Determine an expression for the period of orbit of the ball in terms of $m, \theta, L$ and constants.

Answer: a) When ball is at left acceleration points right




$\vec{a}$

$a_x = a_c$

$a_y = 0$

$$\sum F_x = ma_x \Rightarrow \sum F_x = ma_c$$

$$\sum F_y = ma_y \Rightarrow \sum F_y = 0$$

Now $a_c = v^2/r$ where

r is the radius illustrated.

So $r/L = \sin\theta \Rightarrow r = L\sin\theta$

| | x | y |
|---|---|---|
| $\vec{T}$ | $T\sin\theta$ | $T\cos\theta$ |
| $\vec{F_g}$ | 0 | $-mg$ |

Then

$$\sum F_x = ma_x$$

$$\Rightarrow T\sin\theta = m v^2/r$$

$$\Rightarrow \underbrace{T\sin\theta = \frac{mv^2}{L\sin\theta}}_{①}$$

$$\sum F_y = 0$$

$$\Rightarrow T\cos\theta = mg$$

$$\Rightarrow \underbrace{T = \frac{mg}{\cos\theta}}_{②}$$

Substituting from ② into ① gives

$$\frac{\cancel{m}g}{\cos\theta}\sin\theta = \frac{\cancel{m}v^2}{L\sin\theta} \Rightarrow \boxed{v^2 = gL\frac{\sin^2\theta}{\cos\theta}} \Rightarrow v = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}$$

For a given string length and angle the above formula says that only one speed is possible. However, mass cancels and so multiple masses are possible.

b) $$v = \omega r \Rightarrow (\omega r)^2 = gL\frac{\sin^2\theta}{\cos\theta}$$

$$\Rightarrow \omega^2 r^2 = gL\frac{\sin^2\theta}{\cos\theta}$$

$$\Rightarrow \omega^2 (L\sin\theta)^2 = \frac{gL\sin^2\theta}{\cos\theta}$$

$$\Rightarrow \omega^2 L^2 \cancel{\sin^2\theta} = \frac{gL\cancel{\sin^2\theta}}{\cos\theta} \Rightarrow \omega^2 = \frac{g}{L\cos\theta}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g}{L\cos\theta}}}$$

c) $$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi L\sin\theta}{\sqrt{gL\sin^2\theta/\cos\theta}}$$

$$\Rightarrow T = \sqrt{\frac{4\pi^2 L^2 \cancel{\sin^2\theta}}{gL\cancel{\sin^2\theta}/\cos\theta}} \Rightarrow \boxed{T = \sqrt{\frac{4\pi^2\cos\theta\, L}{g}}}$$