Phys 131 2023

Lecture 17

Mon: HW 5pm

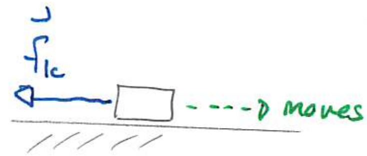
Ex: 130, 131, 132, 134, 135, 136, 139, 141

Tues: Warm Up 7 (D2L)

Friction

We consider two types of friction.

1) Kinetic friction – occurs when one surface slides over the other. Then



   a) the friction force is opposite to motion

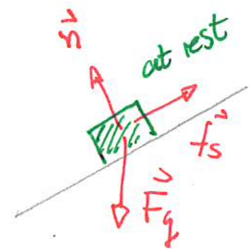
   b) the friction force has magnitude

$$f_k = \mu_k n$$

n → magnitude of normal force

μ_k → coefficient of kinetic friction

2) Static friction – occurs when two surfaces do not slide and is denoted $\vec{f}_s$. Then



   a) direction: opposite to motion that would occur in its absence

   b) magnitude: adjustable up to a maximum

$$f_{s\,max} = \mu_s n \qquad \text{or} \qquad f_s \leq \mu_s n$$

where μ_s = coefficient of static friction

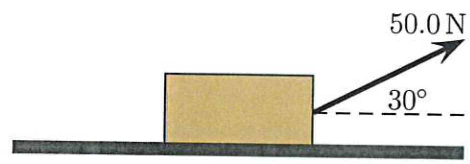
n = magnitude of normal force.

Quiz 1 40% — 70%

10% —

143 Dynamics of a single object with friction

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.250. The primary aim of this exercise will be to determine the acceleration of the box. *(131Sp2023)*



a) Draw a free body diagram for the block.

b) Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \cdots \tag{9}$$

$$F_{\text{net } y} = \Sigma F_y = \cdots \tag{10}$$

Insert as much information as possible about the components of acceleration at this stage. Can you describe in words what these equations are telling you to do?

c) Determine the magnitude of the gravitational force. Let n be the *magnitude* of the normal force. Using this write an expression for the magnitude of the friction force. Do you know the exact number for the friction force at this point?

d) List all the components of all the forces, using one of the two formats below.

$$w_x = \cdots$$
$$w_y = \cdots$$
$$n_x = \cdots$$
$$n_y = \cdots$$
$$\vdots$$

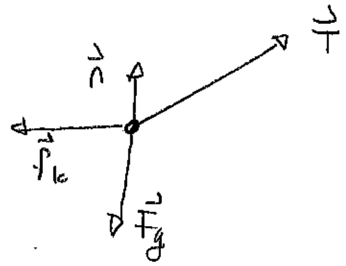
Force	x comp	y comp
$\vec{w}$		
$\vec{n}$		
$\vdots$		

e) Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Does either give the acceleration immediately? Can one of then at least give the normal force immediately?

f) Determine the normal force and use this result to find the acceleration.

g) What tension would be required for the box to have acceleration $4.00\,\text{m/s}^2$ to the right?

Answer: a)

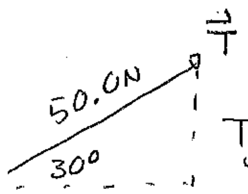


b) $\sum F_x = ma_x$

$\sum F_y = ma_y = 0$

c) $F_g = mg = 15.0\text{kg} \times 9.8\text{m/s}^2 = 147\text{N}$

d)



$T_y = T\sin 30^\circ$

$= 50.0\text{N} \sin 30^\circ$

$= 25.0\text{N}$

$T_x = T\cos 30^\circ$

$= 43.0\text{N}$

	x	y
$\vec{F}_g$	0	$-mg = -147\text{N}$
$\vec{n}$	0	n
$\vec{T}$	$T\cos 30^\circ = 43.0\text{N}$	$T\sin 30^\circ = 25.0\text{N}$
$\vec{f}_k$	$-\mu_k n$	

e) $\sum F_x = ma_x$

$\Rightarrow T\sin 30^\circ - \mu_k n = ma_x \quad \Rightarrow 43.0\text{N} - \mu_k n = ma_x$

$\Rightarrow 43.0\text{N} - 0.250\, n = 15.0\text{kg}\, a_x$

does not give n, a_x immediately

$\sum F_y = ma_y$

$\Rightarrow -mg + n + T\sin 30^\circ = 0$

$\Rightarrow -147\text{N} + n + 25.0\text{N} = 0 \quad \Rightarrow$ gives $n = 147\text{N} - 25.0\text{N} = 122\text{N}$

f) Substitute into $43.0\text{N} - 0.250 n = 15.0\text{kg}\, a_x$

$\Rightarrow 43.0\text{N} - 0.250 \times 122\text{N} = 15.0\text{kg}\, a_x$

$\Rightarrow 12.8\text{N} = 15.0\text{kg}\, a_x$

$\Rightarrow a_x = \dfrac{12.8\text{N}}{15.0\text{kg}} \Rightarrow a_x = 0.853\text{m/s}^2$

g) Here we work symbolically:

$$\sum F_x = ma_x \Rightarrow T\cos 30^\circ - \mu_k n = ma_x \qquad -(A)$$

$$\sum F_y = ma_y \Rightarrow -mg + n + T\sin 30^\circ = 0 \qquad -(B)$$

Then (B) gives $n = mg - T\sin 30^\circ$

and substitute into (A):

$$T\cos 30^\circ - \mu_k(mg - T\sin 30^\circ) = ma_x$$

$$\Rightarrow T\cos 30^\circ + \mu_k \sin 30^\circ T - \mu_k mg = ma_x$$

$$\Rightarrow T\left[\cos 30^\circ + \mu_k \sin 30^\circ\right] = ma_x + \mu_k mg$$

$$= m(a_x + \mu_k g)$$

$$\Rightarrow T = \frac{m(a_x + \mu_k g)}{\cos 30^\circ + \mu_k \sin 30^\circ}$$

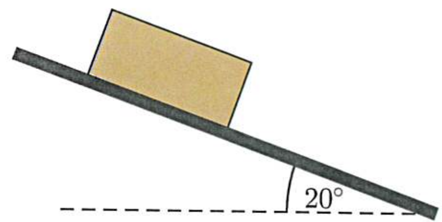
$$= \frac{15\text{kg}\left(4.0\text{m/s}^2 + 0.250 \times 9.8\text{m/s}^2\right)}{\cos 30^\circ + 0.250\sin 30^\circ} = \frac{96.8\text{N}}{0.991}$$

$$\Rightarrow T = 97.7\text{N}$$

Quiz 2 50% – higher { 60% – 80%

149 Speed at the bottom of a rough ramp

A 10 kg box can move along a 4.0 m long rough ramp angled 20° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. The aim of this exercise is to determine the speed of the box at the bottom of the ramp. *(131Sp2023)*



a) Draw a free body diagram for the box.

b) Describe your choice of x and y axes.

c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \cdots \tag{11}$$
$$F_{\text{net } y} = \Sigma F_y = \cdots \tag{12}$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

d) Determine expressions for the *magnitudes* of the gravitational and the friction forces.

e) List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \cdots$$
$$F_{gy} = \cdots$$
$$n_x = \cdots$$
$$n_y = \cdots$$
$$\vdots$$

Force	x comp	y comp
$\vec{\mathbf{F}}_g$		
$\vec{\mathbf{n}}$		
$\vdots$		

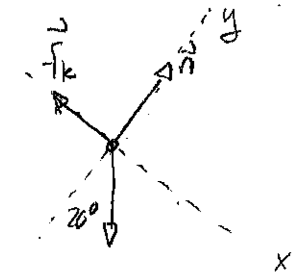
f) Use Eq. (11) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (12). Use the resulting equations to find the acceleration of the box.

g) Determine the speed of the box when it reaches the bottom of the ramp.

h) Do these results depend on the mass of the box?

i) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

Answer: a)



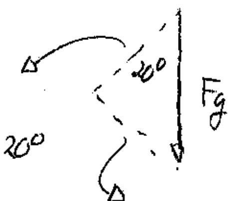
c) $\Sigma F_x = ma_x$

$\Sigma F_y = ma_y$

d) $F_g = mg = 10kg \times 9.8 m/s^2 = 98 N$

$f_k = \mu_k n$

e)



$F_{gy} = -mg\cos 20°$

$= -98 \cos 20°$

$= -92N$

$F_{gx} = mg \sin 20°$

$= 98 \sin 20°$

$= 34N$

	x	y
$\vec{F_g}$	$mg\sin 20°$ $= 34N$	$-mg\cos 20°$ $= -92N$
$\vec{n}$	0	n
f_k	$-\mu_k n$	0

f) $\Sigma F_x = ma_x \Rightarrow mg\sin 20° - \mu_k n = ma_x$

$\Sigma F_y = ma_y \Rightarrow -mg\cos 20° + n = 0 \Rightarrow n = mg\cos 20° = 92N$

Thus $mg\sin 20° - \mu_k mg\cos 20° = ma_x$

$\Rightarrow g\sin 20° - \mu_k g\cos 20° = a_x$

$\Rightarrow g(\sin 20° - \mu_k \cos 20°) = a_x$

$\Rightarrow g(0.107) = a_x \Rightarrow a_x = 1.05 m/s^2$

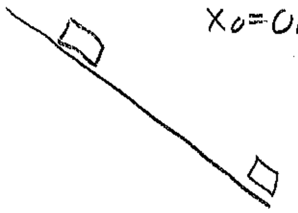
$34N - 0.25 \times 92 = 10kg\, a_x$

$11N = 10kg\, a_x$

$\Rightarrow a_x = 1.1 m/s^2$

rounding difference.

g)



$x_0 = 0m$ $\quad V_{0x} = 0m/s$

$x = 4.0m$ $\quad V_x = ?$

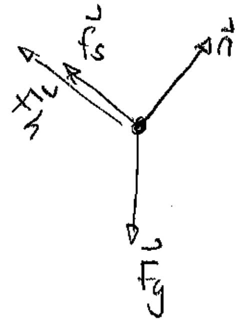
$$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$$

$$= (0m/s)^2 + 2(1.05m/s^2) \times 4.0m = 8.4\ m^2/s^2$$

$$\Rightarrow V_x = \sqrt{8.4m^2/s^2} = 2.9m/s$$

h) No part f) show this.

i)



$\Sigma F_x = ma_x = 0$

$\Sigma F_y = ma_y = 0$

Need max friction $f_s = \mu_s n$

	x	y
$\vec{F_g}$	$+mg\sin 20^\circ$	$-mg\cos 20^\circ$
$\vec{n}$	0	n
$\vec{f_s}$	$-\mu_s n$	0
$\vec{F_h}$	$-F_h$	0

$\Sigma F_y = 0 \Rightarrow n = mg\cos 20^\circ$

$\Sigma F_x = 0 \Rightarrow mg\sin 20^\circ - \mu_s n - F_h = 0$

$$\Rightarrow mg\sin 20^\circ - \mu_s mg\cos 20^\circ - F_h = 0$$

$$\Rightarrow F_h = mg[\sin 20^\circ - \mu_s\cos 20^\circ]$$

$$= mg[\underbrace{\sin 20^\circ - 0.30\cos 20^\circ}_{0.0601}]$$

$$= 5.9N$$