Tues: SPS Meeting, Noon WS 218

Thurs: Discussion / quiz

Ex: 78, 80, 87, 90, 92, 94

Fri:

Circular motion

There are many situations where an object moves in a circle. Examples include

1) object swinging on the end of a string
2) orbital motion of celestial objects.

Demo: The Sky 3D Solar System Simulator

We aim to describe the kinematics of objects moving in such circular trajectories.

Warm Up 1

We can do a preliminary analysis that determines the average acceleration direction.

Quiz 1 70% – 90% ≷ 50% – 70%

Uniform circular motion

An important special case of circular motion is uniform circular motion. Here:

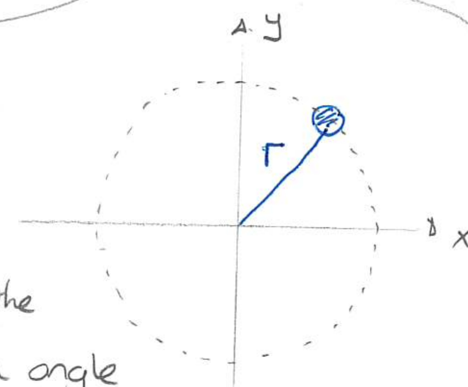
1) the object moves in a circle
2) the speed of the object is constant

The acceleration can be analyzed using

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$x = R\cos(\omega t)$

$y = R\sin(\omega t)$



where ω is the rate at which angle is covered in rad/s and $\omega = v/r$

These produce:

For uniform circular motion the acceleration (centripetal acceleration):

1) points radially inward
2) has magnitude

$$a_c = \frac{v^2}{r}$$

where r = radius of orbit

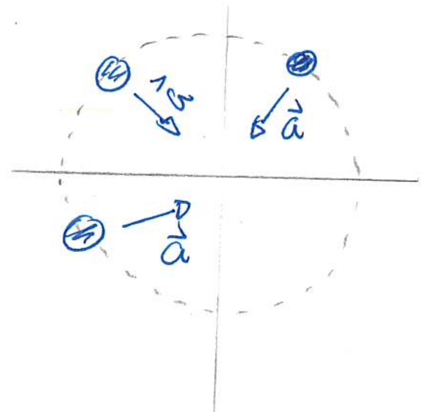
v = speed

Warm Up 2

Quiz 2 80% -> 90% ≷ 90%

Note that for uniform circular motion the magnitude of the acceleration is constant but its direction constantly changes

The diagram illustrates distinct directions.



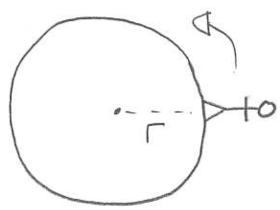
101 Acceleration on Earth's surface

People stand on Earth's surface and are at rest *relative to Earth.* Earth has a radius of 6.4×10^3 km and spins about its poles at a rate of one revolution every 24 hrs. *(131Sp2023)*

a) Determine the acceleration of a person at Earth's equator.

b) Another person stands at a location much closer to the North pole. Is this person's acceleration the same as, larger than or smaller than that of a person at the equator? Explain your answer.

Answer: a) Viewed from the N pole



$r = 6.4 \times 10^3$ km

$r = 6.4 \times 10^6$ m

Need speed. Consider one orbit

$$\text{speed} = v = \frac{\text{distance orbit}}{\text{time taken}}$$

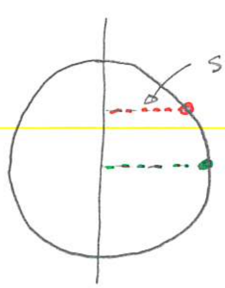
distance orbit ← circumference

time taken ← $24\,\text{hrs} = 24\,\text{hrs} \times \frac{3600\,\text{s}}{1\,\text{hr}} = 86400\,\text{s}$

$$= \frac{2\pi r}{T} = \frac{2\pi \times 6.4 \times 10^6\,\text{m}}{86400\,\text{s}} = 465\,\text{m/s}$$

acceleration: $$a_c = \frac{v^2}{r} = \frac{(465\,\text{m/s})^2}{6.4 \times 10^6\,\text{m}} = 0.034\,\text{m/s}^2$$

b)



smaller r — the radius is smaller ⇒ speed smaller

To check $v = \frac{2\pi r}{T}$

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r^2 / T^2}{r} = \frac{4\pi^2 r}{T^2}$$

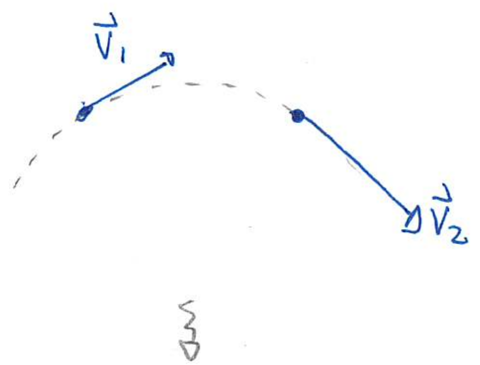
smaller r ⇒ smaller a_c

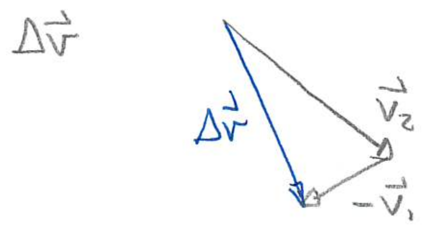
Non-uniform circular motion

An object can speed up or slow down while moving in a circle

Quiz3

In this case





So the acceleration is no longer perfectly inward and has two distinct components