Mon: HW by 5pm

Ex 69, 72, 74, 76

77, 82, 85, 86

Tues: ~~Discussion~~ Warm Up 4

Velocity in Two Dimensions

In two dimensional motion, velocity is a vector determine from the two position co-ordinates

$$\boxed{\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{j}}$$

Thus

$$\boxed{\vec{v} = v_x \hat{i} + v_y \hat{j}}$$

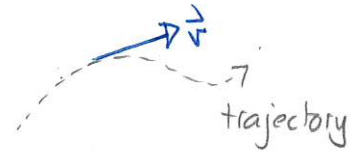
There are two important properties:

1) magnitude

> The speed of the object is the magnitude of the velocity vector
>
> $$v = \sqrt{v_x^2 + v_y^2}$$
>
> speed

2) direction

> The direction of the velocity vector is tangent to the trajectory in the direction of motion



Demo: PhET Ladybug 2D

Vectors → show velocity

Motion → Ellipse Trace vector

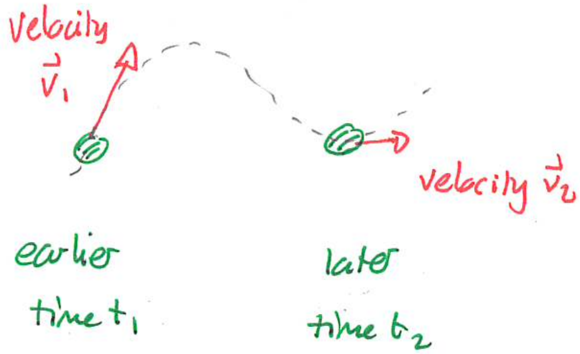
Quiz 1 80% - 95% 70% - 90%

Quiz 2 70% - 80% 40% - 50%

Acceleration in Two Dimensions

Acceleration is again the rate at which the velocity vector changes
A preliminary definition is average acceleration over an interval.

Observe object at two instants

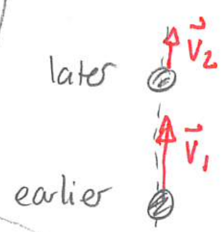


$\rightsquigarrow$ The average acceleration from t_1 to t_2 is

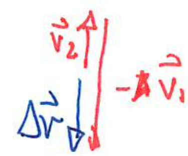
$$\vec{a}_{average} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Example: A ball is thrown vertically upward. Determine the direction of the acceleration vector as it moves up wards after leaving the hand.

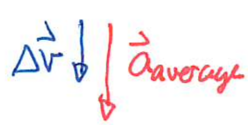
Answer: ① Sketch trajectory and velocity vectors at earlier + later moments



$\rightarrow$ ② Find $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$



$\rightarrow$ ③ Since $\Delta t > 0$ direction of $\vec{a}_{average}$ is same as direction of $\Delta\vec{v}$



Quiz3 50% $\{$ 40% - 70%

The more precise definition is.

> The acceleration of an object is
> $$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Note that

1) acceleration is a vector

$$\boxed{\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath} \quad \text{where} \quad a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt}}$$

2) acceleration has a direction. This is the same as the direction of the change in velocity. It may or may not coincide with the direction of motion.

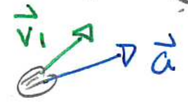
To illustrate this consider the situation where <u>acceleration is constant</u>. Then

$\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ exactly. This implies

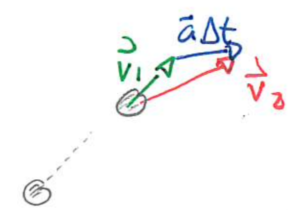
$$\Delta\vec{v} = \vec{a}\,\Delta t \quad \Rightarrow \quad \vec{v}_2 - \vec{v}_1 = \vec{a}\,\Delta t \Rightarrow \vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t$$

So

Knowing $\vec{v}_1$ at an earlier instant and $\vec{a}$ subsequently



$\rightarrow$ Determine $\vec{v}_2$ at a later instant via $\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t$

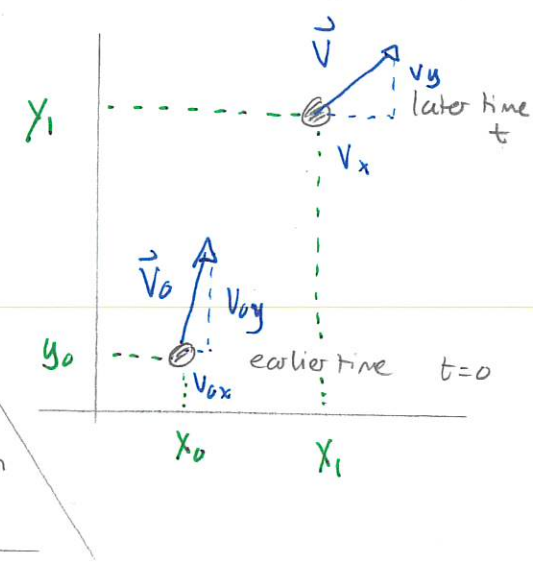


In this example the object speeds up and curves more to the right. Specifically

$$\vec{v} = \vec{v}_0 + \vec{a}\,t$$

$$\Rightarrow \quad V_x = V_{0x} + a_x t$$

$$V_y = V_{0y} + a_y t$$



We see that

> If the acceleration is constant then the horizontal + vertical components of the motion can be analyzed independently

Using the symbols of the diagram:

Horizontal Only	Vertical Only
$V_x = V_{ox} + a_x t$	$V_y = V_{oy} + a_y t$
$X = X_o + V_{ox} t + \frac{1}{2} a_x t^2$	$y = y_o + V_{oy} t + \frac{1}{2} a_y t^2$
$V_x^2 = V_{ox}^2 + 2a_x (X - X_o)$	$V_y^2 = V_{oy}^2 + 2a_y (y - y_o)$

time is the only common variable.

Projectile motion

A projectile is an object that only moves under the influence of Earth's gravity.

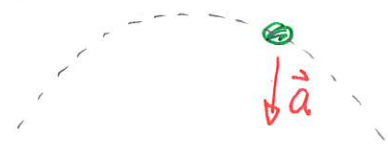
Demo: PhET Projectile Motion

Experiments show

The acceleration of a projectile is constant with components

$a_x = 0 \, m/s^2$

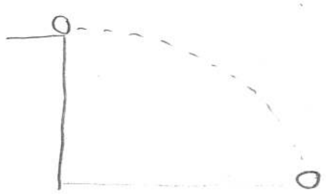
$a_y = -g = -9.8 \, m/s^2$



83 Running off a roof

A person runs with speed 8.0 m/s off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land. *(131Sp2023)*

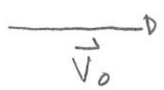
a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground.



List as many of the variables as possible. Use the format:

$t_0 =$ 0 s	$t =$
$x_0 =$ 0 m	$x =$?
$y_0 =$ 3.0 m	$y =$ 0.0 m
$v_{0x} =$ 8.0 m/s	$v_x =$
$v_{0y} =$ 0 m/s	$v_y =$
$a_x =$ 0 m/s^2	$a_y = -9.8$ m/s^2

b) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{\mathbf{v}}_0$. Enter these in the list above.



$V_{ox} = 8.0$ m/s $\quad V_{oy} = 0$ m/s

c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it.

Need X

$X = \cancel{X_o} + V_{ox} t + \frac{1}{2} \cancel{a_x} t^2$

$X = V_{ox} t$ we need t.

You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?

d) Use the vertical aspects of the object's motion to solve for this other unknown variable and use this result to answer the question of this problem.

$$\overset{0}{\cancel{y}} = y_0 + \overset{0}{\cancel{V_{oy}t}} + \frac{1}{2}a_y t^2$$

$$0\,\text{m} = 3.0\,\text{m} + \frac{1}{2}(-9.8\,\text{m/s}^2)\,t^2$$

$$-3.0\,\text{m} = -4.9\,\text{m/s}^2\,t^2$$

$$t^2 = \frac{3.0\,\text{m}}{4.9\,\text{m/s}^2} = 0.61\,\text{s}^2 \Rightarrow t = \sqrt{0.61\,\text{s}^2}$$

$$t = 0.78\,\text{s}$$

$$x = V_{ox}\,t$$

$$= 8.0\,\text{m/s} \times 0.78\,\text{s}$$

$$= 6.3\,\text{m}$$

Suppose that the person ran and jumped from the building at an angle of 30° above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?

e) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{\mathbf{v}}_0$. Reconstruct the list of variables for the problem.

f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.

g) Use the kinematic equations to determine the maximum height that the person reaches.