Thurs: Discussion / Quiz

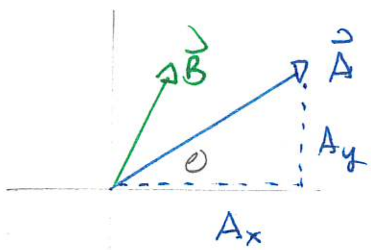
Ex: 57, 58, 59, 64, 65, 66, 67, 68

Fri:

Vector algebra with components

Vector algebra is most conveniently accomplished via vector components.

Given two vectors $\vec{A}, \vec{B}$



want $\vec{D} = \alpha\vec{A} + \beta\vec{B}$

Get components, e.g.

$A_x = A\cos\theta$

$A_y = A\sin\theta$

insert signs as appropriate.

$D_x = \alpha A_x + \beta B_x$

$D_y = \alpha A_y + \beta B_y$

Reconstruct $\vec{D}$ from its components

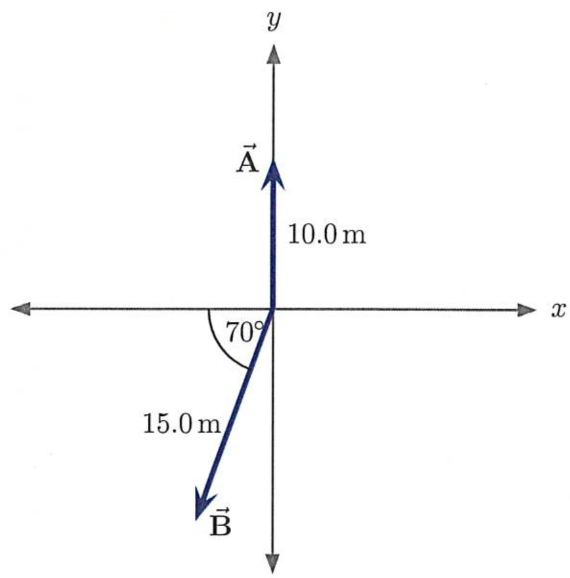
then $D = \sqrt{D_x^2 + D_y^2}$

$\theta = \arctan(D_y / D_x)$

configure as appropriate.

62 Vector addition: algebraic method, 1

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. *(131Sp2023)*



a) Determine the components of $\vec{\mathbf{A}}$.

b) Determine the components of $\vec{\mathbf{B}}$.

c) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

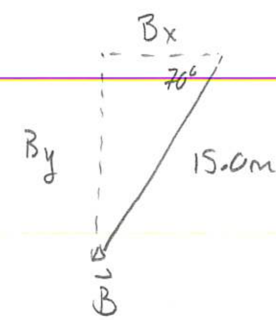
d) Determine the magnitude of $\vec{\mathbf{C}}$.

Answer: a) $A_x = 0\,m$

$A_y = 10.0\,m$

b) $B_x = -B\cos 70^\circ$

$= -15.0\,m \cos 70^\circ = -5.1\,m$

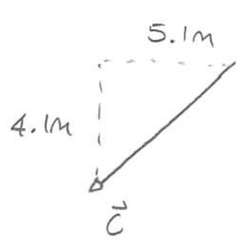


$B_y = -B\sin 70^\circ$

$= -15.0\,m \sin 70^\circ = -14.1\,m$

c) $C_x = A_x + B_x = 0\,m - 5.1\,m = -5.1\,m$

$C_y = A_y + B_y = 10.0\,m - 14.1\,m = -4.1\,m$



$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-5.1\,m)^2 + (-4.1\,m)^2}$

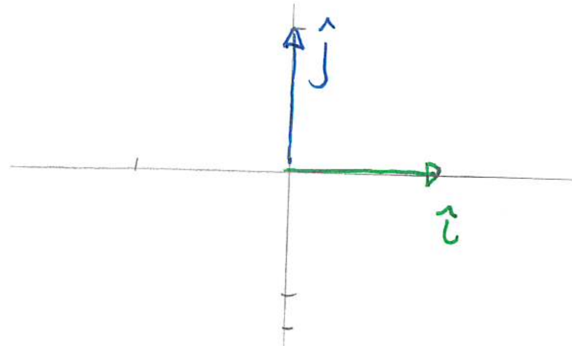
$= 6.5\,m$

Unit vectors

A vector is completely specified by listing its components. These can be used reconstruct the vector along with special unit vectors. In two dimensions these are:

$\hat{i}$ = unit vector along x-axis

$\hat{j}$ = unit vector along y-axis



Then any vector can be expressed as.

Any vector $\vec{A}$ in two dimensions has form

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

where A_x = x-component of $\vec{A}$, A_y = y-component of $\vec{A}$ — could be positive or negative

Slide 1

Slide 2

Slide 3

In the previous exercise

$$\vec{A} = 0.0\text{m}\,\hat{i} + 10.0\text{m}\,\hat{j} = 10.0\text{m}\,\hat{j}$$

$$\vec{B} = -5.1\text{m}\,\hat{i} - 14.1\text{m}\,\hat{j}$$

$$\vec{A}+\vec{B} = 10.0\text{m}\,\hat{j} - 5.1\text{m}\,\hat{i} - 14.1\text{m}\,\hat{j} = -5.1\text{m}\,\hat{i} + (10.0\text{m} - 14.1\text{m})\,\hat{j}$$

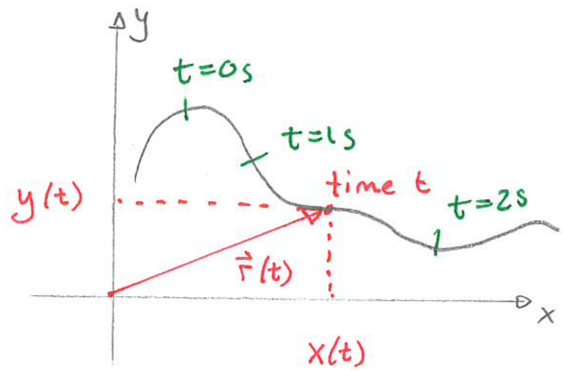
$$= -5.1\text{m}\,\hat{i} - 4.1\text{m}\,\hat{j}$$

Warm Up 1

Kinematics in two dimensions

We need to adapt the language of one-dimensional kinematics to describe two-dimensional situations.

Consider an object moving in the x-y plane. We can represent the trajectory via



$x(t)$ = horizontal position

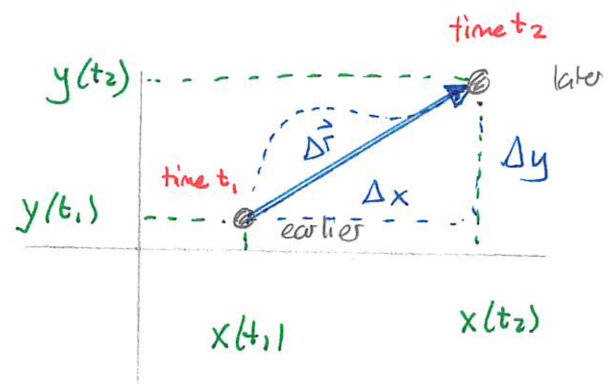
$y(t)$ = vertical position.

In formal treatments, these are combined into a single position vector

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

and we do formal manipulations with this. However, we can construct velocity directly using $x(t), y(t)$

This begins with average velocity, which is constructed as:



Observe the object at two instants, t_1 and t_2. Record

	earlier time t_1	later time t_2
horiz. pos	$x(t_1)$	$x(t_2)$
vert. pos	$y(t_1)$	$y(t_2)$

Form displacement vector

$$\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath}$$

where $\Delta x = x(t_2) - x(t_1)$

$\Delta y = y(t_2) - y(t_1)$

The average velocity vector from time t_1 to t_2 is

$$\vec{v}_{average} = \frac{\Delta x}{\Delta t}\,\hat{\imath} + \frac{\Delta y}{\Delta t}\,\hat{\jmath}$$

Warm up 2

Quiz 1 80% → 100% } 95%
Quiz 2 95% }

In order to capture details of the velocity at one instant we need to look over very small time intervals. We then get

The instantaneous velocity of an object is a vector

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{j}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

Note that :

1) velocity is a vector

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

where the components are

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

2) the direction of the velocity vector $\vec{v}$ is tangent to the trajectory in the direction of motion

Quiz 3

3) the instantaneous speed is the magnitude of the velocity vector