Tues: Warm Up 3 (D2L) by 9am

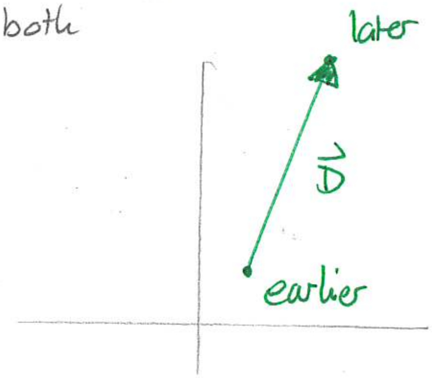
Thurs: Discussion/quiz

Ex:

Vector Addition

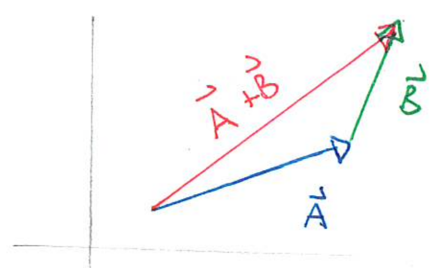
A displacement vector is an arrow with both

1) magnitude
2) direction



This indicates a change in position. We now construct processes for adding such vectors. The conceptual idea will be:

* given two vectors $\vec{A}, \vec{B}$
* consider successive displacements, first, along $\vec{A}$ and, second, along $\vec{B}$
* create the single direct displacement. This is $\vec{A}+\vec{B}$



Demo: PhET Vector Addition → Explore 2D Tab

* create two vectors, show sum.

More precisely

> Given displacement vectors $\vec{A}, \vec{B}$, then $\vec{A}+\vec{B}$ is the single displacement vector
> * starting at tail of $\vec{A}$
> * ending at head of B , provided head $\vec{A}$ and tail $\vec{B}$ coincide.

Quiz 1 60% - 80% $\rightsquigarrow$ 60% - 90%

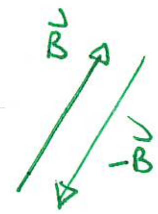
We see that it will not generally be true that if

$$\vec{C} = \vec{A} + \vec{B} \qquad \text{then} \qquad C = A + B$$

vectors   magnitudes

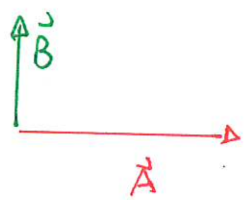
A related operation is vector subtraction. This requires the negative of a vector:

> If $\vec{B}$ is any vector then
> $-\vec{B}$ is another vector with the same magnitude and opposite direction

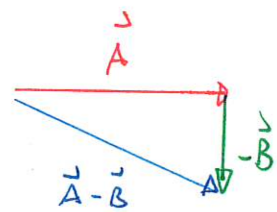


Then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



and



Quiz 2 30% - 80% $\rightsquigarrow$ 40% - 80%

Scalar multiplication

A vector can be rescaled by multiplying with a scalar (number)

> Let $\vec{A}$ be any vector and c any number. Then $c\vec{A}$ is a vector with
> 1) magnitude $|c| A$
> 2) direction $= \begin{cases} \text{same as } \vec{A} & \text{if } c > 0 \\ \text{opposite to } \vec{A} & \text{if } c < 0 \end{cases}$

Quiz 3 20 - 60% $\rightsquigarrow$ 60% $\to$ 90%

Vector components

Graphical methods for adding vectors can convey concepts but they are not precise nor suitable for mathematics. We need a method for determining the sum that only uses algebraic techniques. One strategy involves representing vectors in terms of components.

Any vector $\vec{A}$ in two dimensions $\xrightarrow{\text{equivalent}}$ Two real numbers A_x, A_y

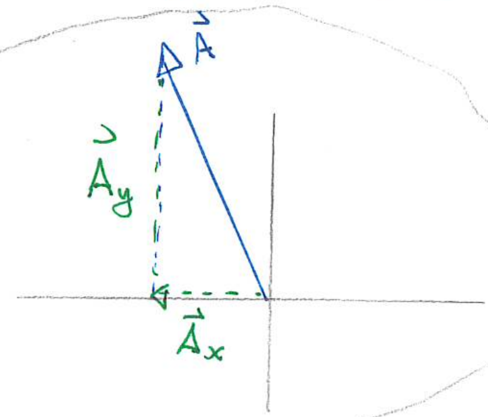
Demo: PhET Vector addition — Tab explore 2D

Show single vector with * component vectors

* component numbers.

The scheme is

Given a vector $\vec{A}$



Construct two component vectors

$\left.\begin{array}{l}\vec{A}_x \text{ - purely horizontal} \\ \vec{A}_y \text{ - purely vertical}\end{array}\right\}$ such that $\vec{A} = \vec{A}_x + \vec{A}_y$

The components of $\vec{A}$ are two real numbers

Horizontal component A_x

* magnitude equal to magnitude of $\vec{A}_x$

* positive if $\vec{A}_x \rightarrow$

* negative if $\vec{A}_x \leftarrow$

Vertical component A_y

* magnitude equal to magnitude of $\vec{A}_y$

* positive if $\vec{A}_y \uparrow$

* negative if $\vec{A}_y \downarrow$

Then one can show:

If $\vec{D} = \vec{A} + \vec{B} + \vec{C} + \dots$ then

$$D_x = A_x + B_x + C_x + \dots$$

$$D_y = A_y + B_y + C_y + \dots$$

If $\vec{D} = c\vec{A}$ where c is a number

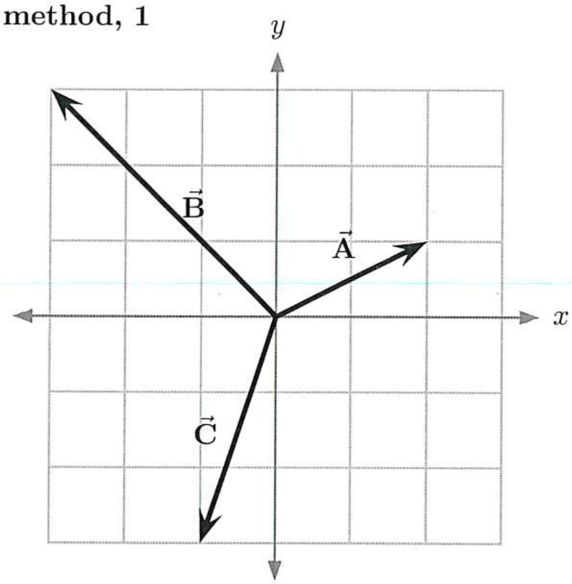
$$D_x = c A_x$$

$$D_y = c A_y$$

We can do vector algebra by working with components.

60 Vector addition: graphical and algebraic method, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. *(131Sp2023)*

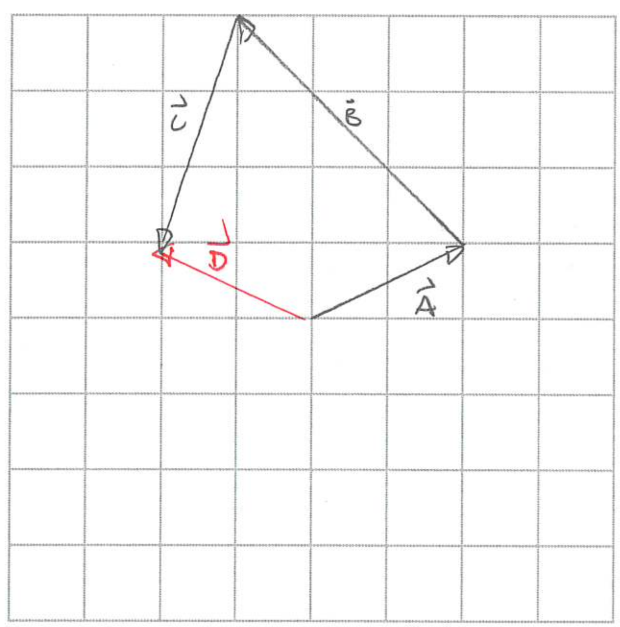


a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head–to–tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

b) List the horizontal and vertical components of each of $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ and use these components to determine the components of $\vec{\mathbf{D}}$. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

a)

$D = \sqrt{2^2+1^2} = \sqrt{5}$



b) $A_x = 2$ $\quad A_y = 1$

$B_x = -3$ $\quad B_y = 3$

$C_x = -1$ $\quad C_y = -3$

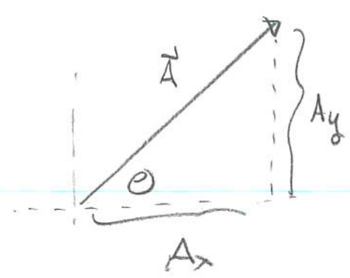
$D_x = A_x + B_x + C_x = 2 - 3 - 1 = -2$

$D_y = A_y + B_y + C_y = 1 + 3 - 3 = 1$

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{5}$

Calculating vector components

We can calculate vector components using trigonometry



Then A = magnitude of A (hypotenuse)

$\Rightarrow A_x = A\cos\theta$

$A_y = A\sin\theta$

Thus

Given $\vec{A}, \vec{B}$ want $\vec{D} = \alpha\vec{A} + \beta\vec{B}$

α, β real

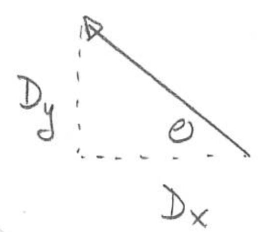
$\rightarrow$

$D_x = \alpha A_x + \beta B_x$

$D_y = \alpha A_y + \beta B_y$

gives components of $\vec{D}$

reconstruct $\vec{D}$ from components



$\rightarrow$

magnitude $D = \sqrt{D_x^2 + D_y^2}$

direction via angle, e.g.

$\theta = \arctan D_y / D_x$