Mon: HW by 5pm

- * return on paper
- * counts 12pts (out of eventual total 600pts)
- * do not use Internet resources.

Exercises: 3, 7, 11, 15, 16, 18, 17, 20

Tues: Warm Up by 9am

LABS MEET

Instantaneous velocity.

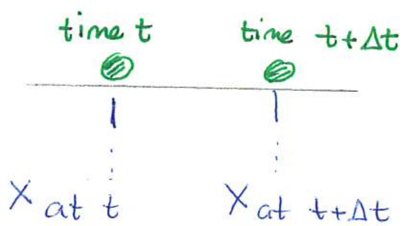
Average velocity is not good at capturing information about motion over an extended interval. In physics this is replaced by instantaneous velocity or velocity

(instantaneous) velocity ⇝ rate at which position changes at one instant.

We define this via

The (instantaneous) velocity of an object at time t is the limiting value of the average velocity over the interval $t \to t+\Delta t$ as $\Delta t \to 0$. Thus

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x_{\text{at } t+\Delta t} - x_{\text{at } t}}{\Delta t}$$



units: m/s

In practice we can determine velocity via:

t	x
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given data for

x vs t

$\Downarrow$

Do limiting calculations

Given position as a function of time, use calculus

e.g.

$$x = -2t + \frac{1}{2}t^2$$

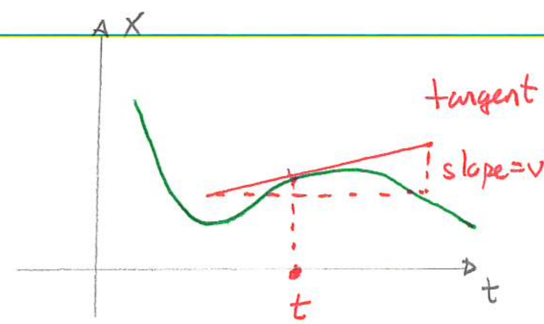
$\Downarrow$ calculus

$$v = -2 + t$$

There is an additional useful interpretation:

Given a graph of position vs time, the velocity at time t is

v = slope of tangent to graph of x vs t at time t



Warm Up 1

Note that the instantaneous speed is:

$\boxed{\text{Instantaneous speed} = s = \text{magnitude of velocity} = |v|}$

Quiz 1 70% → 90% $\Downarrow$ 50% – 60%

Quiz 2

Note the two parts of velocity:

magnitude (of velocity)

$\Downarrow$

speed

AND

sign of velocity

$\Downarrow$

direction

● → v > 0

← ● v < 0

Position from velocity

We are often given velocity information and need to extract position information from this. A special case is uniform motion

"UNIFORM MOTION ONLY"

Given velocity v $\Rightarrow$ $\bar{v} = v$ $\Rightarrow$ $v = \dfrac{\Delta x}{\Delta t}$ $\Rightarrow$ $\boxed{\Delta x = v \Delta t}$

displacement

So we can get the displacement over a time interval via simple algebra.

Fundamental Mechanics: Group Exercise 1

27 January 2023

Names: ________________ ________________ ________________

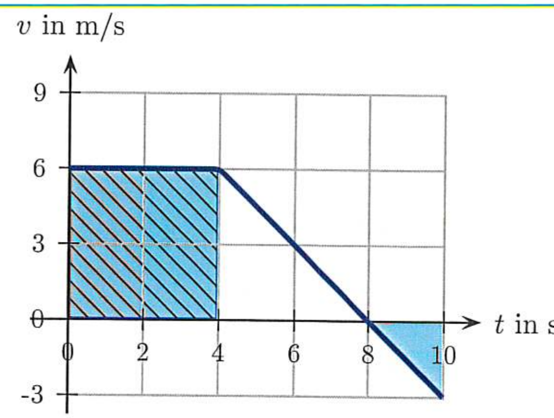
________________ ________________

1 Rotating object

Take a rectangular object with three sides of different lengths: a phone is a good example. Try to flip the phone in such a way that it rotates and does not "tumble." Try this for three distinct axes. Is it easier to to this about some axes rather than others? Have you ever noticed this before? Where? *The instructor will provide some assistance.*

2 Crawling slug

A slug crawls along a straight wire, starting at $x = 0.0\,\text{m}$ at $t = 0.0\,\text{s}$. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. *(131Sp2023)*



a) Determine the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$.

b) How is the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$ related to the shaded area *between the graph and the horizontal axis* ($v = 0.0\,m/s$)?

c) Assuming that the answer to the previous question is true in general, determine the displacement of the slug from $t = 4.0\,\text{s}$ to $t = 8.0\,\text{s}$.

d) Is the displacement of the slug from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$ positive or negative? How might this relate to the shaded area from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$?

Answer: a) $\Delta x = v\Delta t \Rightarrow \Delta x = 6\,m/s \times 4s = 24m$

b) The area is height × length = $6m/s \times 4s = 24\,m$. It's the same

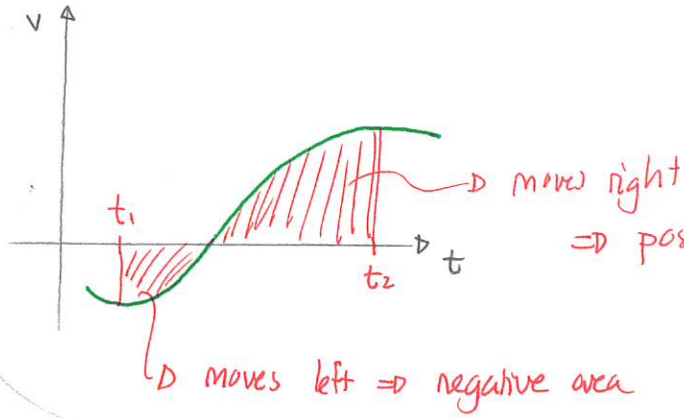
c) The area here is $\frac{1}{2}$ base × height = $\frac{1}{2}\, 4s \times 6m/s = 12m \Rightarrow 12m$

d) It's moving left, so x decreases $\Rightarrow$ displacement negative $\Rightarrow$ negative of area.

In general

Given a graph of velocity vs time $\rightarrow$ Displacement from time t_1 to t_2 is

$$\boxed{\Delta x = \text{area between graph and axis from time } t_1 \text{ to } t_2}$$



Calculating velocity

If we know x as a function of t then calculus provides:

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \text{"derivative of } x \text{ with respect to } t\text{"}}$$

Rules for calculating derivatives include

$$\boxed{\text{If } x = a t^n \text{ where } a, n \text{ are constants then } \frac{dx}{dt} = n a t^{n-1}}$$

Example: Suppose $x = (5.0\,m/s^2)\, t^2 + (3.0\,m/s)\, t$. Determine velocity at 3s

Answer: $v =$ derivative of x w.r.t t

$= \text{deriv}\left(5\,m/s^2\, t^2\right) + \text{derive}\left(3.0\,m/s\ t\right)$

$= 2 \times 5\,m/s^2\, t + 1 (3.0\,m/s)\, \underbrace{t^{1-1}}_{=1}$

$= 10\,m/s^2\, t + 3.0\,m/s$

At $t = 3s$ $\quad v = 10\,m/s^2 \times 3s + 3.0\,m/s = 33\,m/s$