Phys 131
Spring 2023

Fundamental Mechanics: Class Exam 3

21 April 2023

Name: _____ **Total:** /70

Instructions

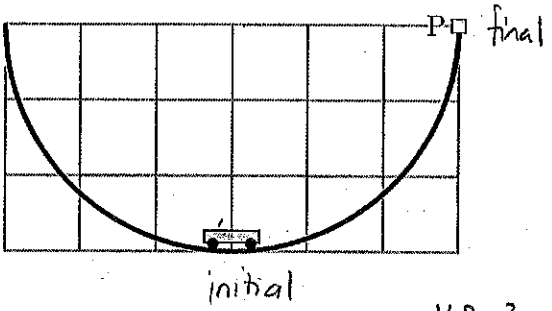
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$g = 9.80\,\mathrm{m/s^2}$

Question 1

A 6.0 kg cart can slide along a semi-circular track with radius 4.0m. The track is oriented vertically and at the lowest point the cart is has velocity 7.0 m/s to the right. Determine whether the cart will reach the top of the track. Ignore friction and air resistance.



$E_f = E_i$ will give max height

$K_f + U_{gf} = K_i + U_{gi}$

$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$ ($v_f = 0$)

$gy_f = \frac{1}{2}v_i^2$

$y_f = \frac{v_i^2}{2g} = \frac{(7.0\,m/s)^2}{(2 \times 9.8\,m/s^2)} = 2.5m$

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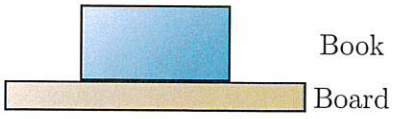
The edge is at 3.0m so it does not reach P.

$\Delta\vec{r}$ ↑ ↑ $\vec{n}$

Question 2

A book rests on the top of a board. The board is moved vertically up; the book is always in contact with the board. Which of the following (choose one) is true of the work, W, done by the normal force acting on the book?



i) $W = 0$ in all circumstances.

ii) $W > 0$ in all circumstances.

iii) $W < 0$ in all circumstances.

iv) $W > 0$ if the book is slowing down, $W < 0$ if the book is speeding up.

v) $W < 0$ if the book is slowing down, $W > 0$ if the book is speeding up.

$W = n\Delta r\cos 0^\circ = n\Delta r > 0$

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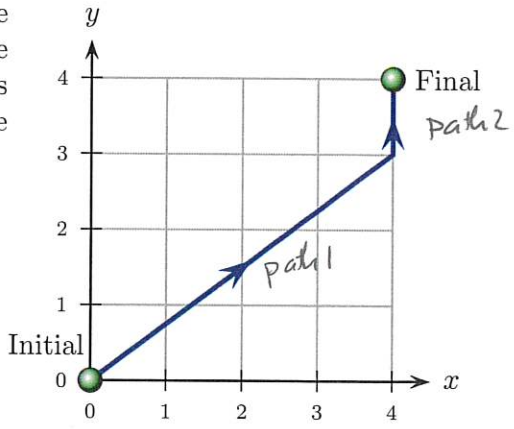
Question 3

A ball moves along the indicated path. Throughout the motion a hidden object exerts a force $\vec{\mathbf{F}} = 6\,\mathrm{N}\,\hat{\mathbf{j}}$ on the ball. Determine the work done by this force on the ball as it moves from the indicated initial to final location. The graph grid units are meters.



Add the work for both paths.

$W = W_{path\,1} + W_{path\,2}$

(+1) $W_{path\,1} = \vec{F} \cdot \Delta\vec{r}_1$

here $\vec{F} = 0\,N\,\hat{i} + 6\,N\,\hat{j}$

$\Delta\vec{r}_1 = 4m\,\hat{i} + 3m\,\hat{j}$ } $= 0$ $W_{path\,1} = 0\times 4 + 6N\times 3m = 18J$

(~~+4~~) (+5)

$W_{path\,2} = \vec{F} \cdot \Delta\vec{r}_2$

$\Delta\vec{r}_2 = 0m\,\hat{i} + 1m\,\hat{j}$ $= 0$ $W_{path\,2} = 0\times 4 + 1m\times 6N = 6J$

(~~+4~~) (+3)

$W = 18J + 6J = 24J$

(+1)

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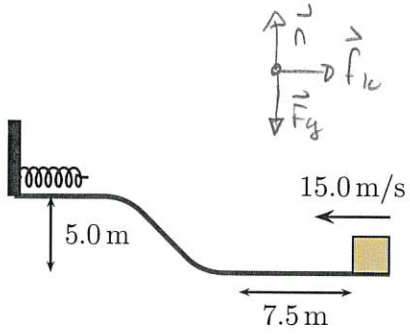
Question 4

A 4.0 kg block can slide along the illustrated track. At an initial moment it moves with speed 15.0 m/s to the left as illustrated. As it passes along the lower horizontal section it traverses as 7.5 m long section where there is a 12 N kinetic friction force. The track beyond that section is frictionless and it approaches a spring, whose spring constant is 600 N/m, and which is attached to a fixed wall. Ignore air resistance.



a) Determine the speed of the block immediately before it reaches the spring.

$\Delta E = W_{nc} \Rightarrow E_f = E_i + W_{nc}$ (+1)

$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 + W_{fric}$

each (+1)

$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + W_{fric}$

$x_f = 0m \quad x_i = 0m$

$y_f = 5.0m \quad y_i = 0m$

$v_f = ? \quad v_i = 15 m/s$

$\frac{1}{2}4kg\,v_f^2 + 4.0kg \times 9.8 m/s^2 \times 5.0m = \frac{1}{2}4.0kg \times (15 m/s)^2 - 90J$ (+1)

Then $W_{fric} = f_k \Delta r \cos\theta$ (+1)

$= 12N \times 7.5m \cos 180^\circ$

$= -90J$ (+2)

$2.0kg\,v_f^2 + 196J = 450J - 90J$

$\Rightarrow 2.0kg\,v_f^2 = 164J$

$v_f^2 = 82 m^2/s^2 \Rightarrow v_f = \sqrt{82 m^2/s^2} = 9.1 m/s$ (+4)

b) Determine the maximum spring compression as the spring slows the block.

$E_f = E_i$ (+1)

$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$

(+1) → $v_f = 0 m/s \quad v_i = 9.1 m/s$

$x_f = ? \quad x_i = 0m$

$y_f = 5.0m \quad y_i = 5.0m$

$\Rightarrow \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i - mgy_f$

(+4)

$\Rightarrow x_f^2 = \frac{m}{k}v_i^2$

$\Rightarrow x = \sqrt{\frac{m}{k}}\,v_i = \sqrt{\frac{4.0kg}{600N/m}}\,v_i$

$= 0.74m$

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Question 5

Two identical cannonballs are fired from the ground with the same speed. Cannonball A is fired vertically upward while cannonball B is fired at an angle of 55° above the horizontal. Ignore air resistance in the following.

$E_f = E_i \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$

$\Rightarrow v_f = v_i$ regardless

a) Which of the following (choose one) is true?

i) Cannonball A hits the ground with the same speed as cannonball B.

ii) Cannonball A hits the ground with a larger speed than cannonball B.

iii) Cannonball A hits the ground with a smaller speed than cannonball B.

b) Which of the following (choose one) is true at the *highest point in each ball's trajectory?*

i) Cannonball A has the same speed as cannonball B.

ii) Cannonball A has a larger speed than cannonball B.

iii) Cannonball A has a smaller speed than cannonball B.

Cannonball A has $v = 0$

Cannonball B has $v_x \neq 0$

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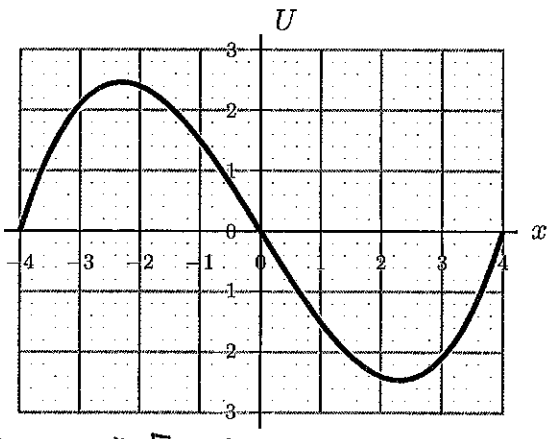
Question 6

A particle that can move along the x axis is subjected to the illustrated potential energy U (horizontal axes grid units are meters). Is the force on the particle at $x = 0\,\text{m}$ bigger than, smaller than or the same as the force at $x = -2.2\,\text{m}$? Explain your answer.



$F_x = -\frac{dU}{dx} = -$ slope of U vs x

At $x = -2.2\,\text{m}$ the slope is almost zero $\Rightarrow F \approx 0$

At $x = 0\,\text{m}$ the slope is beyond -1. $\Rightarrow F > 1$

So the force is larger at $x = 0$.

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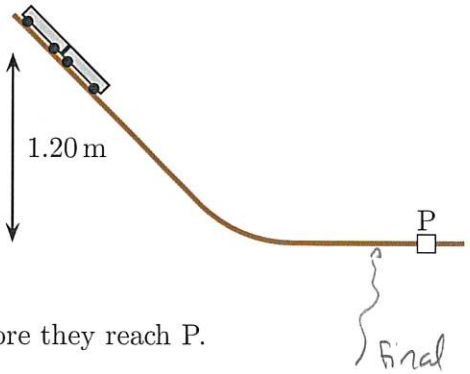
Question 7

Two identical carts, each with mass 0.600 kg, can slide down a track. They are released from rest at the indicated height and slide together at the same speed down the track. When they reach the location marked P the track is horizontal and a spring between the carts deploys. This separates the carts. After this, the cart on the left moves left with speed 3.00 m/s. Ignore friction and air resistance in this situation.



a) Determine the speed of the carts immediately before they reach P.

$\Delta E = 0$ (since $W_{nc} = 0$)

$y_i = 1.20\,m$ $y_f = 0\,m$

$v_i = 0\,m/s$ $v_f =$

+1 $E_f = E_i$

$K_f + U_{gf} = K_i + U_{gi}$

$\frac{1}{2}Mv_f^2 + Mgy_f = \frac{1}{2}Mv_i^2 + Mgy_i$

+1 +1

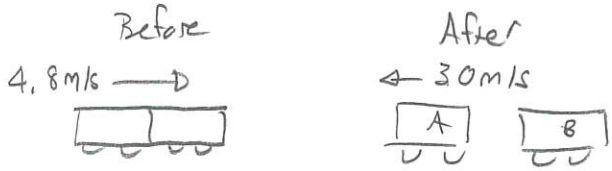
$\Rightarrow \frac{1}{2}v_f^2 = gy_i$

$\Rightarrow v_f^2 = 2gy_i \Rightarrow v_f = \sqrt{2gy_i}$

$= \sqrt{2 \times 9.8\,m/s^2 \times 1.20\,m}$

$= 4.85\,m/s$ +4

Question 7 continued ...



b) Determine the speed of the cart on the right after the two carts have separated.

Net external force $= 0 \Rightarrow \vec{p}_{tot}$ conserved.

$p_{tot\,f} = p_{tot\,i}$] +1

$mv_{Af} + mv_{Bf} = 2m v_i$

+1

$v_{Bf} = -v_{Af} + 2v_i$

+2

$= -(-3.0\,m/s) + 2 \times 4.85\,m/s$] +3

$= 12.7\,m/s$

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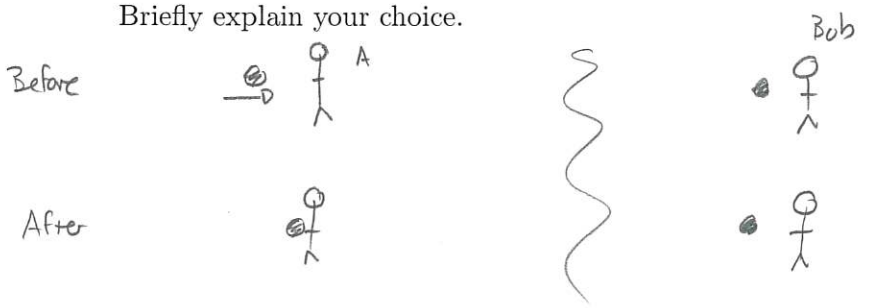
Question 8

Two people, Alice and Bob, of equal mass, sit at rest on a sheet of ice. Identical balls are thrown east with the same speed to each person. Alice catches and holds the ball thrown to her. Bob bats the ball thrown to him directly back so that it reverses direction. Which of the following (choose one) is true after these events?

i) Alice and Bob remain at rest.
ii) Alice and Bob move east with the same speed as each other.
iii) Alice moves east with a speed greater than that of Bob.
iv) Alice moves east with a speed smaller than that of Bob.

Briefly explain your choice.



$p_{tot\,f} = p_{tot\,i}$ same

$p_{bf} + p_{Af} = p_{Bi}$ $\quad$ $p_{bf} + p_{Bch\,f} = p_{Bi}$

positive $\quad$ negative

$\Rightarrow p_{Bch\,f}$ is greater

$\Rightarrow p_{Af}$ is larger.

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