

Fundamental Mechanics: Class Exam 1

23 September 2022

Name: _____

Total: /70

Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.81 \text{ m/s}^2$$

Question 1

A ball slides in a straight line along a flat surface with constant acceleration. At one moment it is at rest and it travels 1.8m in the next 3.0s. Determine the acceleration of the ball.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad \neq 2 + 1$$

$$\text{If } v = \frac{\Delta x}{\Delta t} \text{ only } 2/6$$

$$1.8 \text{ m} = \cancel{0 \text{ m}} + \cancel{0 \text{ m/s} \times 3.0 \text{ s}} + \frac{1}{2} a_x (3.0 \text{ s})^2$$

$$\Rightarrow 1.8 \text{ m} = 4.5 \text{ s}^2 a_x \quad \Rightarrow a_x = \frac{1.8 \text{ m}}{4.5 \text{ s}^2} = 0.40 \text{ m/s}^2 \quad \left. \vphantom{\frac{1.8 \text{ m}}{4.5 \text{ s}^2}} \right\} +4$$

/6

Question 2

A ball, attached to a stretchy string, falls vertically down, stretching the string. The ball eventually reverses direction and moves vertically up. Which of the following (choose one) is true of the ball's acceleration *at its lowest point*? Use coordinates where upward is positive.

i) $a = 0 \text{ m/s}^2$

ii) $a > 0 \text{ m/s}^2$

iii) $a < 0 \text{ m/s}^2$ but $a \neq -9.8 \text{ m/s}^2$

iv) $a = -9.81 \text{ m/s}^2$

Just before

Just after

$$v_1 < 0$$

$$v_2 > 0$$

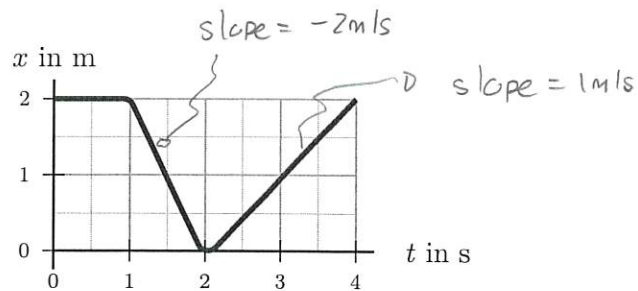
$$\Delta v = v_2 - v_1 > 0$$

$$\Rightarrow a = \frac{\Delta v}{\Delta t} > 0$$

/5

Question 3

An ant walks back and forth along a perfectly straight stick. The position vs. time graph for the ant during a period of its motion is illustrated. In the following, consider right as positive.



- a) Is the ant's speed at 1.5s the same, larger, or smaller than that at 3.0s? Explain your answer.

$$\text{speed} = |\text{velocity}| \quad \text{velocity} = \text{slope} \times \text{vs}t \quad +1$$

$$\text{so at } 1.5s \quad \text{speed} = 2m/s \quad +1 \quad \left. \vphantom{\text{so at } 1.5s} \right\} \Rightarrow \text{larger at } 1.5s \quad +1$$

$$3.0s \quad \text{speed} = 1m/s \quad +1$$

- b) Is the average acceleration of the ant from 1.5s to 3.0s zero, positive or negative? Explain your answer.

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{1m/s - (-2m/s)}{1.5s} = 2.0m/s^2 \quad +2$$

This is positive +1

/8

Question 4

A 0.30 m diameter wheel rotates at a constant rate about an axle through its center. A snail sticks to a point halfway from the axle to the rim of the wheel. It takes 1.7s for the wheel to complete one rotation.

- a) Determine the snail's speed. $+2$

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{t} = \frac{2\pi \cdot 0.075m}{1.7s} = 0.28m/s \quad \left. \vphantom{\text{speed}} \right\} +3 +2$$

orbital radius = $\frac{1}{2}$ wheel radius = 0.075m

- b) Determine the snail's acceleration.

$$a_c = \frac{v^2}{r} = \frac{(0.28m/s)^2}{0.075m} = 1.0m/s^2$$

+1 +2

/7

Question 5

An atom moving in a straight line passes a mark, moving right with speed 40 m/s. It subsequently moves with constant acceleration to the right in a straight line and stops a distance of 8.0×10^{-2} m from its initial location.

a) Determine the acceleration of the atom.

mark
→
⊖

stop
⊖

$$t_0 = 0$$

$$t =$$

$$x_0 = 0 \text{ m}$$

$$x = 8.0 \times 10^{-2} \text{ m}$$

$$v_0 = 40 \text{ m/s}$$

$$v = 0 \text{ m/s}$$

$$a = ??$$

+3

+1

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(0 \text{ m/s})^2 = (40 \text{ m/s})^2 + 2a(8.0 \times 10^{-2} \text{ m})$$

+4
+1

$$\Rightarrow 0 \text{ m}^2/\text{s}^2 = 1600 \text{ m}^2/\text{s}^2 + 16 \times 10^{-2} \text{ m } a$$

$$\Rightarrow a = -\frac{1.6 \times 10^3 \text{ m}^2/\text{s}^2}{1.6 \times 10^{-1} \text{ m}} \Rightarrow a = -1.0 \times 10^4 \text{ m/s}^2$$

b) Determine the time taken for the atom to stop.

$$v = v_0 + at \Rightarrow 0 \text{ m/s} = 40 \text{ m/s} + (-1.0 \times 10^4 \text{ m/s}^2) t$$

+1

$$\Rightarrow -40 \text{ m/s} = -1.0 \times 10^4 \text{ m/s}^2 t$$

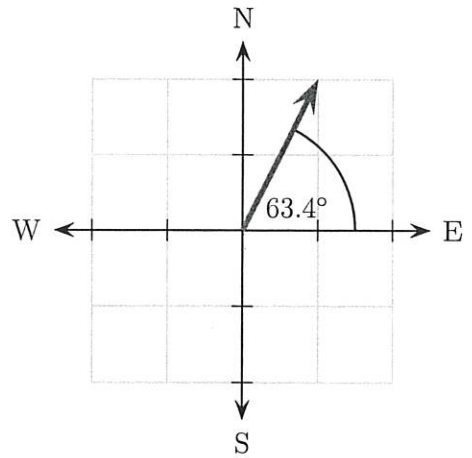
$$\Rightarrow t = \frac{40 \text{ m/s}}{1.0 \times 10^4 \text{ m/s}^2} = 4.0 \times 10^{-3} \text{ s}$$

+3

/12

Question 6

A crab scuttles around a horizontal piece of sand. First the crab moves at an angle 63.4° North of East (direction indicated in the diagram) for 25 cm. Then the crab moves directly south (S) for a distance of 35 cm and finally directly west (W) for a distance of 20 cm. Determine how far the crab is from its starting point at the end of these three stages. The diagram indicates the directions N, S, E, W.



There are three displacements:

$$\vec{A} = \begin{array}{l} 25\text{cm} \\ \nearrow \\ 63.4^\circ \end{array}$$

$$\vec{B} = \begin{array}{l} \downarrow \\ 35\text{cm} \end{array}$$

$$\vec{C} = \begin{array}{l} \leftarrow \\ 20\text{cm} \end{array}$$

Adding
vectors
(+2)

The distance is the magnitude of the displacement

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

To do this use components: $\vec{A} = A_x \hat{i} + A_y \hat{j}$

$$A_x = A \cos 63.4^\circ$$

$$= 25\text{cm} \cos 63.4^\circ$$

$$= 11.2\text{cm}$$

$$A_y = A \sin 63.4^\circ$$

$$= 25\text{cm} \sin 63.4^\circ$$

$$= 22.4\text{cm}$$

(+3) So $\vec{A} = 11.2\text{cm} \hat{i} + 22.4\text{cm} \hat{j}$

$$\vec{B} = -35\text{cm} \hat{j}$$

$$\vec{C} = -20\text{cm} \hat{i}$$

$$\Rightarrow \vec{D} = 11.2\text{cm} \hat{i} + 22.4\text{cm} \hat{j} - 35\text{cm} \hat{j} - 20\text{cm} \hat{i}$$

$$= -8.8\text{cm} \hat{i} - 12.6\text{cm} \hat{j}$$

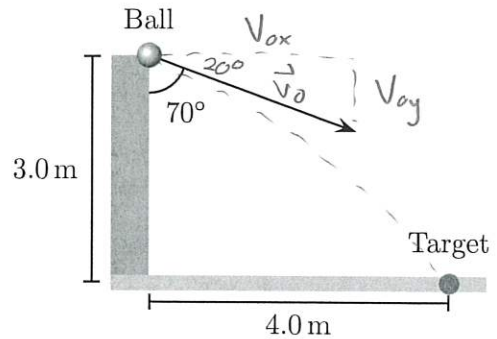
(+2) Then $D = \sqrt{(-8.8\text{cm})^2 + (-12.6\text{cm})^2} = \sqrt{237\text{cm}^2}$

$$= 15\text{cm}$$

/10

Question 7

A person on a wall and attempts to throw a ball at a target on the ground below. The ball leaves the person's hand at a height of 3.0 m from the ground. The target is 4.0 m from the base of the wall. The ball is aimed so that it leaves the wall initially traveling with speed 10 m/s at an angle of 70° from the wall. Determine whether the ball passes above the target and, if so, how high it is when it is directly above the target or, if not, how far it falls short of the target.



Find out what vertical position is when $x = 4.0\text{m}$. If it is negative then the ball falls short. We need components of v_0

(+3) [

$$v_{0x} = v_0 \cos 20^\circ = 10\text{m/s} \cos 20^\circ = 9.4\text{m/s}$$

$$v_{0y} = -v_0 \sin 20^\circ = -10\text{m/s} \sin 20^\circ = -3.4\text{m/s}$$

$$t_0 = 0\text{s} \quad t = ?$$

$$x_0 = 0\text{m} \quad x = 4.0\text{m}$$

$$y_0 = 3.0\text{m} \quad y = ??$$

$$v_{0x} = 9.4\text{m/s}$$

$$v_{0y} = -3.4\text{m/s}$$

$$a_x = 0\text{m/s}^2 \quad a_y = -9.8\text{m/s}^2$$

(+1)
(+2)

(+1) [$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\Rightarrow 4.0\text{m} = 9.4\text{m/s}t \Rightarrow t = 0.43\text{s}$

(+2) (+3)

(+1) [$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$$= 3.0\text{m} - 3.4\text{m/s} \times 0.43\text{s} + \frac{1}{2}(-9.8\text{m/s}^2)(0.43\text{s})^2$$

$$= 0.66\text{m}$$

(+1) (+3)

(+2) [

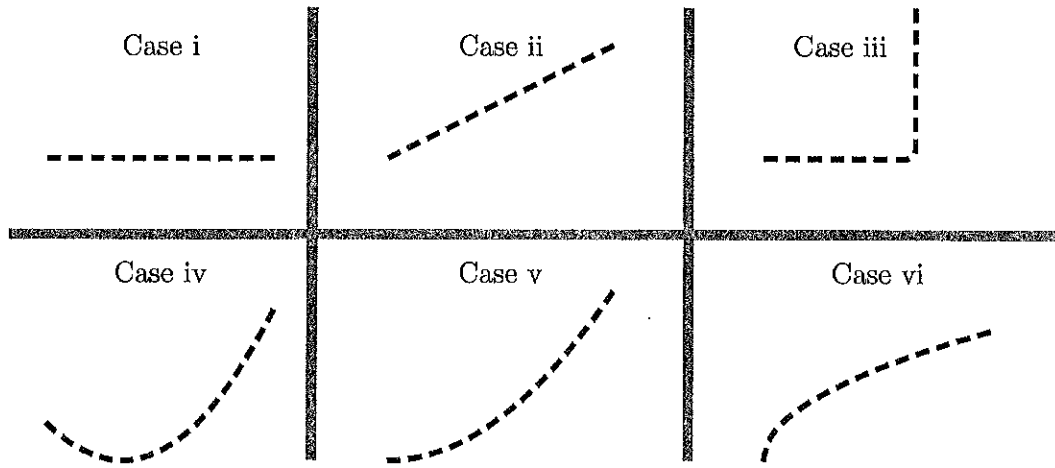
This is positive, so the ball passes 0.66m above the target.

Question 8

A hockey puck can slide on a smooth horizontal surface. At an initial instant, its velocity, \vec{v}_0 , viewed from above is as illustrated. It is equipped with a propulsion device such that, at all later times, it accelerates with a constant acceleration, \vec{a} , as illustrated.



Which of the following best represents the trajectory of the puck from the initial instant onwards? **Explain your answer.**



In general $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = \vec{a} \Delta t$

So initially $\vec{v}_0 = \rightarrow$

After 1s $\vec{v}_1 = \vec{v}_0 + \vec{a} \Rightarrow \vec{v}_1 =$

After 2s $\vec{v}_2 = \vec{v}_1 + \vec{a} \Rightarrow \vec{v}_2 =$

The puck angles upward

In case iv there is a velocity earlier and this is not possible

\Rightarrow Case v