Phys 131
Fall 2022

Fundamental Mechanics: Final Exam v2

14 December 2022

Name: Solution **Total:** /150

Instructions

- There are 16 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

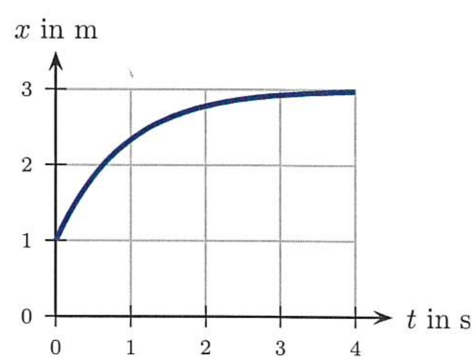
$g = 9.80\,\text{m/s}^2$ $G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

Also on v1

Question 1 ✓

A cart is launched horizontally. A graph of position versus time is illustrated.



a) Which of the following (choose one) is true regarding the velocity of the cart during the period for which the motion is graphed?

i) $v \geqslant 0$ at all times.

ii) $v \leqslant 0$ at all times.

iii) At some times $v > 0$ and at others $v < 0$.

b) Which of the following (choose one) is true regarding the acceleration of the cart during the period for which the motion is graphed?

i) $a = 0$ at all times.

ii) $a \geqslant 0$ at all times.

iii) $a \leqslant 0$ at all times.

iv) At some times $a > 0$ and at others $a < 0$.

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Question 2 ✓

a) An aircraft travels horizontally in a straight line 200 m above the ground and with constant speed of 100 m/s. A ball is dropped from the airplane. Determine the time taken for the ball to hit the ground. Ignore air resistance.

initial

final

$t_0=0$ $t=?$

$x_0=0$ $x=$

$y_0=200m$ $y=0m$

$V_{0x}=100m/s$

$V_{0y}=0m/s$

$a_x=0m/s^2$

$a_y=-9.8m/s^2$ +1

$$\cancel{y}=y_0+\cancel{v_{0y}}t+\tfrac{1}{2}a_yt^2 \quad +1$$

$$0=y_0+\tfrac{1}{2}a_yt^2$$

$$0=200m+\tfrac{1}{2}(-9.8m/s^2)t^2$$

$$\Rightarrow \sqrt{\frac{200m \times 2}{9.8m/s^2}}=t \quad \Rightarrow t=6.4s$$

b) At the instant that the ball hits the ground is the aircraft directly above the ball, ahead of the ball or behind the ball? Explain your answer.

The horizontal component of the balls velocity stays constant +3
and matches the aircraft $\Rightarrow$ aircraft directly above

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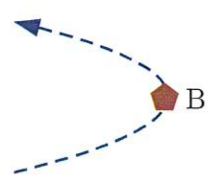
Question 3 ✓

A snail moves at a constant speed on a horizontal surface. Its trajectory as viewed from above is illustrated. Which of the following (choose one) regarding the snail's instantaneous acceleration, $\vec{a}$, at point B is true?



i) $\vec{a} = 0$.

ii) $\vec{a} \neq 0$ and points $\downarrow$.

iii) $\vec{a} \neq 0$ and points $\longrightarrow$.

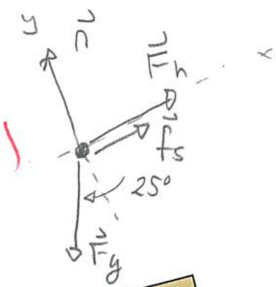
iv) $\vec{a} \neq 0$ and points $\longleftarrow$.

$\vec{V}_f$ $\vec{V}_i$

$\Delta\vec{v} = -\vec{V}_i$ $\vec{V}_F$ $\Delta\vec{v}$

/5

$\Rightarrow \vec{a}$ is $\leftarrow$

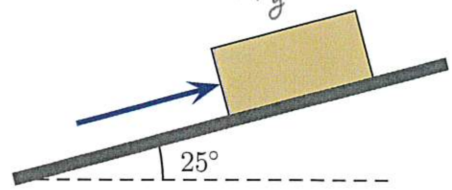


(+1)

tilted axes

Question 4 ✓

A 50 kg box lies on an inclined ramp. The coefficient of static friction between the box and the ramp is 0.18. You push parallel to the ramp against the block.



coeff of kinetic friction is 0.10

a) Determine the minimum force with which you must push to keep the box at rest.

$\sum F_x = ma_x = 0$
$\sum F_y = ma_y = 0$ } at rest (+1)

$F_g = mg$ (+1)

$f_s \leq \mu_s n$ (+1)

	x	y (+3)
$\vec{F}_g$	$-mg\sin 25°$	$-mg\cos 25°$
$\vec{n}$	0	n
$\vec{f}_s$	f_s	0
F_h	F_h	0

$\sum F_y = 0 \Rightarrow n - mg\cos 25°$

$\Rightarrow n = mg\cos 25°$] (+2)

(+5)

$\sum F_x = 0 \Rightarrow -mg\sin 25° + f_s + F_h = 0$

$\Rightarrow F_h = mg\sin 25° - f_s$

$\Rightarrow F_h \geq mg\sin 25° - \mu_s n$

$\Rightarrow F_h \geq mg\sin 25° - \mu_s mg\cos 25°$

$\Rightarrow F_h \geq mg(\sin 25° - \mu_s \cos 25°) = 50\text{kg} \times 9.8\text{ m/s}^2 (\sin 25° - 0.18\cos 25°)$

$= 127\text{N}$

$\Rightarrow F_h \geq 127\text{N}$

b) The entire situation is repeated except that the block slides down the ramp at a constant speed. Will the minimum force produced by the hand to do this be different to what it would be to keep the block at rest? Explain your answer.

(+2)

The acceleration is again zero. However there is now kinetic friction and this force is smaller. So the hand force would be larger.

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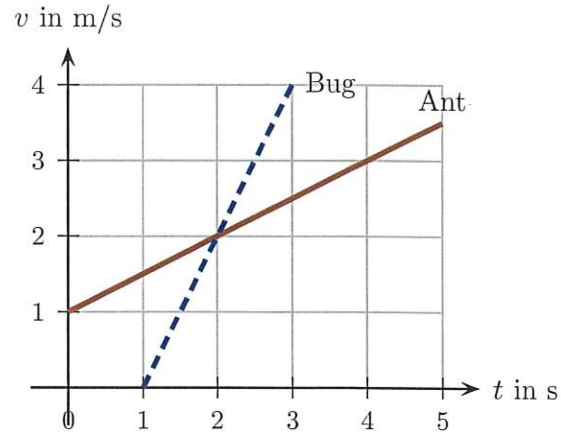
Question 5

An ant and a bug walk along straight wires. The graph illustrates their velocities vs. time. They have the same mass. Which of the following (choose one) is true at 2.0 s?



i) The net force on the ant is the same as on the bug.

ii) The net force on the ant is larger than the net force on the bug.

iii) The net force on the ant is smaller than the net force on the bug.

Briefly explain your answer.

$\vec{F}_{net} = m\vec{a}$

$F_{net} = ma$

Bug $a = \frac{\Delta v}{\Delta t} = 2\,m/s^2$

Ant $a = \frac{\Delta v}{\Delta t} = 0.5\,m/s^2$

Ant smaller accel $\Rightarrow$ force smaller

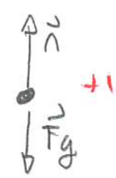
/6

Question 6

A 0.40 kg object slides around the inside a frictionless vertical hoop with radius 0.90 m. The speed of the object at the bottom of the hoop is ~~3.0~~ ~~6.0~~ ~~0.30~~ m/s. Determine the normal force exerted by the hoop on the object at the bottom.

change later

Acceleration is up and $a = \frac{v^2}{r}$ (+1)



$\sum F_y = ma_y$ (+1)

$\Rightarrow \underbrace{n - mg}_{(+2)} = mv^2/r$

$\Rightarrow n = mg + \frac{mv^2}{r} = m\left(g + \frac{v^2}{r}\right) = m\left(9.8\,m/s^2 \times \frac{(0.3\,m/s)^2}{0.90\,m}\right)$

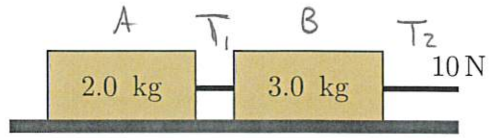
(+1)

$= 0.40\,kg \times 9.9\,m/s^2$

$= 3.96\,N$

/6

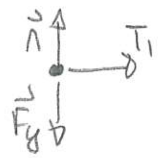
Question 7



Two blocks, connected by a rope, and with indicated masses can move along a frictionless horizontal surface. The blocks are initially at rest and then a rope pulls right with tension 10 N.

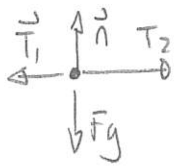
a) Determine the acceleration of the blocks while the rope pulls and use this to determine the tension in the rope connecting the blocks.

On A



$\sum F_x = M_A a \Rightarrow T_1 = M_A a$

On B



$\sum F_x = M_B a \Rightarrow T_2 - T_1 = M_B a$

Adding gives $T_2 = (M_A + M_B) a \Rightarrow a = \dfrac{T_2}{M_A + M_B} \Rightarrow a = \dfrac{10N}{5.0kg}$

$\Rightarrow a = 2.0 m/s^2$

Then $T_1 = M_A a$

$\Rightarrow T_1 = \dfrac{M_A}{M_A + M_B} T_2 = \dfrac{2 kg}{5 kg} \times 10N \Rightarrow T_1 = 4.0N$

b) Suppose that the blocks were flipped with the 2.0 kg block on the right, the 3.0 kg block on the left and the rope pulling to the right with 10 N as before. Would the tension in the connecting rope be larger than, smaller than or the same as that for the original arrangement? Explain your answer.

Yes according to this it will be larger when M_A is larger.

(+3)

/16

Same as V1 **Question 8**

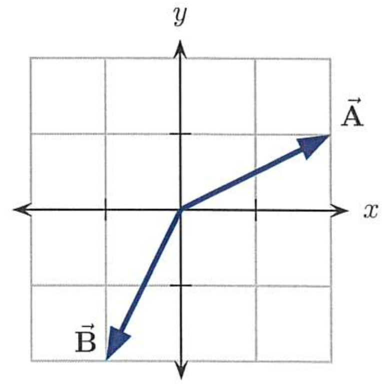
Two balls are fired with the same speeds from a bridge above a lake. Ball A is fired horizontally and ball B is is fired at an angle of 30° below the horizontal. Both eventually hit the lake. Which of the following (choose one) is true at the instant just before the balls hit the lake? Ignore air resistance.

i) The speed of ball A is the same as that of B.
ii) The speed of ball A is smaller than that of B.
iii) The speed of ball A is larger than that of B.

/5

Question 9

Determine the dot product, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ of the two illustrated vectors. The axis units are in meters.



$\vec{A} = 2\hat{i} + \hat{j}$ +1 $\quad \vec{B} = -\hat{i} - 2\hat{j}$ +1

$\vec{A} \cdot \vec{B} = 2(-1) - 1(2) = -4$

/6

Same as V1 **Question 10**

A person and sled (combined mass 100 kg) move along a horizontal icy surface with speed 2.0 m/s. The person holds a 20 kg rock which initially moves moves along with the sled. The person subsequently throws the rock horizontally and after this the person and sled are at rest. Determine the speed with which the rock must be thrown for this to happen.

/6

Question 11

Two 0.20 kg blocks can slide along the illustrated frictionless surface. The right end of the track is 1.5 m higher than the left end. Block B is initially at rest to the left of a spring which is relaxed and which has spring constant 100 N/m. The right end of the spring is fixed against a rigid wall. Block A is launched to the right with speed 20 m/s and it collides with and sticks to block B. The two subsequently move together.

a) Determine the speed of the blocks immediately after the collision.

+4

Momentum conserved

$p_{tot\,f} = p_{tot\,i}$

$V_{Ai} = 20\,m/s$ $\quad V_{Af} = V_{Bf} = V_f$

$V_{Bi} = 0\,m/s$

$m_A V_{Af} + m_B V_{Bf} = m_A V_{Ai} + \cancel{m_B V_{Bi}}$

$\Rightarrow (m_A + m_B) V_f = m_A V_{Ai}$

$\Rightarrow V_f = \dfrac{m_A}{m_A + m_B} V_{Ai} = \dfrac{1}{2} \times 20\,m/s = 10\,m/s$

b) Determine the maximum compression of the spring after the blocks collide with it.

+12

Energy conserved.

$y_i = 0 \quad y_f = 1.5\,m$

$E_f = E_i$

$\Delta x_i = 0 \quad \Delta x_f = ??$

$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi}$

$V_i = 10\,m/s \quad V_f = 0$

$\cancel{\frac{1}{2} M v_f^2} + M g y_f + \frac{1}{2} k (\Delta x_f)^2 = \frac{1}{2} M v_i^2 + \cancel{M g y_i} + \cancel{\frac{1}{2} k (\Delta x_i)^2}$ $\quad M = $ total Mass $= 0.40\,kg$

$\Rightarrow \frac{1}{2} k (\Delta x_f)^2 = \frac{1}{2} M v_i^2 - M g y_f$

$\Rightarrow \frac{1}{2}\, 100\,N/m\, (\Delta x_f)^2 = \frac{1}{2}\, 0.40\,kg \times (10\,m/s)^2 - 0.40\,kg \times 9.8\,m/s^2 \times 1.5\,m$

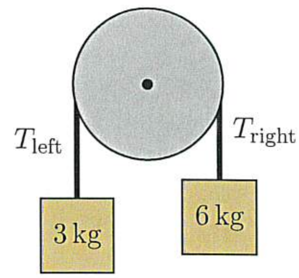
$50\,kg/s^2 (\Delta x_f)^2 = 14.12\,kg\,m^2/s^2$

$\Rightarrow (\Delta x_f) = 0.53\,m$

/16

Same as V, **Question 12** ✓

Two blocks are connected by a massless string that runs over a 4 kg pulley. The pulley can rotate about a frictionless axle and, when it does this, the string does not slip. Let T_{left} be the tension in the string on the left and T_{right} the tension in the string on the right. Which of the following (choose one) is true?



i) $T_{\text{right}} = T_{\text{left}}$ regardless of the blocks' motion.

ii) $T_{\text{right}} > T_{\text{left}}$ regardless of the blocks' motion.

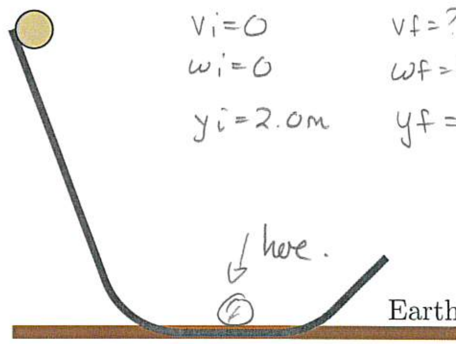
iii) $T_{\text{right}} < T_{\text{left}}$ regardless of the blocks' motion.

iv) Whether T_{right} is larger or smaller depends on whether the block on the right moves up or down.

/5

Question 13 ✓

A 0.250 kg solid uniform cylinder with radius 0.10 m is held at rest on a track as illustrated. At this instant, the bottom of the cylinder is exactly 2.0 m above the Earth. The cylinder is released and rolls without slipping along the track. The end of the track is at a height of 0.40 m from the Earth. Determine the maximum speed of the cylinder as it moves along the track.



$v_i = 0$ $\quad$ $v_f = ?$

$\omega_i = 0$ $\quad$ $\omega_f = ?$

$y_i = 2.0\,\text{m}$ $\quad$ $y_f = 0\,\text{m}$

$E_f = E_i$ ← ? (+1)

~~(+2)~~ (+2)

$K_{\text{trans}f} + K_{\text{rot}f} + U_{gf} = K_{\text{trans}i} + K_{\text{rot}i} + U_{gi}$

+4

$\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + \cancel{m g y_f} = \cancel{\frac{1}{2} m v_i^2} + \cancel{\frac{1}{2} I \omega_i^2} + m g y_i$

(+1) ~~(+1)~~ (+1) (+1)

For a cylinder $I = \frac{1}{2} M R^2$ and no slip $\Rightarrow \omega_f = V/R$ (+2)

$\Rightarrow \frac{1}{2} \cancel{m} v_f^2 + \frac{1}{2}\,\frac{1}{2} \cancel{M} R^2 v_f^2 / R^2 = \cancel{m} g y_i$

(+2)

+2

$\Rightarrow \left(\frac{1}{2} + \frac{1}{4}\right) v_f^2 = g y_i$

$\Rightarrow V_f^2 = \frac{4}{3} g y_i \Rightarrow V_f = \sqrt{\frac{4}{3} \times 9.8\,\text{m/s}^2 \times 2.0\,\text{m}}$

$= 5.1\,\text{m/s}$

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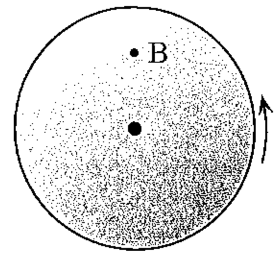
Absuvi

Question 14

A 40 kg solid disk with radius 2.0 m rotates counterclockwise about an axle through its center. At an initial moment the disk rotates with angular 40 rad/s. A brake pressing on the disk applies a constant kinetic frictional force at the point labeled B and stops the disk 10 s after the initial moment. The distance from B to the center of the disk is 1.5 m. The mass in the disk is uniformly distributed.



a) Determine the angular acceleration of the disk.

b) Determine the magnitude of the frictional force exerted by the brake on the disk.

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A15v1 **Question 15**

Two satellites, P and Q, orbit Earth in circles. They are the same distance from Earth. The mass of P is ten times the mass of Q. Let a_P be the magnitude of the acceleration of P and a_Q be the magnitude of the acceleration of Q. Assume that the only force acting on each satellite is Earth's gravity. Which of the following (choose one) is true?

i) $a_Q = \frac{1}{10} a_P$

ii) $a_Q = a_P$

iii) $a_Q = 10 a_P$

/5

A16v1 **Question 16**

A 2000 kg satellite orbits Jupiter (mass 1.9×10^{27} kg and radius 7.0×10^{7} m) at a constant speed in a circle. The distance from the center of Jupiter to the satellite is 2.0×10^{9} m. **Starting with and using Newton's Second Law,** derive an expression for the satellite's speed and use this to determine the time taken to complete one orbit.

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