Fundamental Mechanics: Final Exam v1

12 December 2022

Name: SOLUTION **Total:** /150

Instructions

- There are 16 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

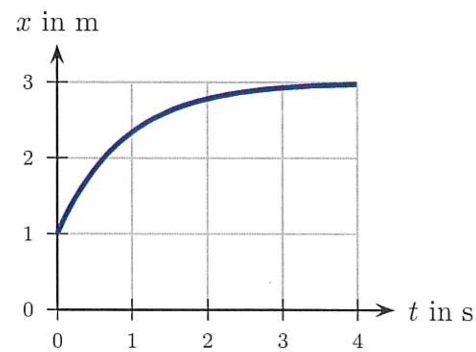
Physical constants and useful formulae

$g = 9.80\,\text{m/s}^2$ $\quad G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$ $\quad$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ $\quad$ Hollow sphere: $I = \frac{2}{3}MR^2$ $\quad$ Solid sphere: $I = \frac{2}{5}MR^2$

Question 1

A cart is launched horizontally. A graph of position versus time is illustrated.



a) Which of the following (choose one) is true regarding the velocity of the cart during the period for which the motion is graphed?

i) $v \geqslant 0$ at all times.
ii) $v \leqslant 0$ at all times.
iii) At some times $v > 0$ and at others $v < 0$.

b) Which of the following (choose one) is true regarding the acceleration of the cart during the period for which the motion is graphed?

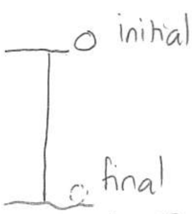
i) $a = 0$ at all times.
ii) $a \geqslant 0$ at all times.
iii) $a \leqslant 0$ at all times.
iv) At some times $a > 0$ and at others $a < 0$.

v = slope x vs t
slope positive $\Rightarrow V \geqslant 0$
slope decreasing with time $\Rightarrow \frac{dv}{dt} \leq 0 \Rightarrow a \leq 0$

/8

Question 2

a) A ball is dropped from rest at the edge of a horizontal table. The table edge is 1.5 m above the ground. Determine the time taken for the ball to reach the ground. Ignore air resistance.



$t_0 = 0s$ $\quad t = ??$

$y_0 = 1.5m$ $\quad y = 0m$

$V_{0y} = 0m/s$ $\quad V_y = ?$

$a = -g = -9.8m/s^2$

(+1)

(+1)

$\cancel{y} = y_0 + \cancel{v_{0y}} t + \frac{1}{2} a_y t^2$

$\Rightarrow 0 = y_0 + \frac{1}{2} a_y t^2$

$\Rightarrow -y_0 = \frac{1}{2} a_y t^2$

$\Rightarrow \frac{-2y_0}{a_y} = t^2$

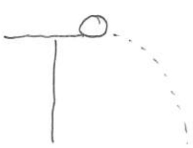
$\Rightarrow t = \sqrt{\frac{2y_0}{g}}$

$= \sqrt{\frac{2 \times 1.5M}{9.8m/s^2}}$

$\Rightarrow t = 0.55s$

(+5)

b) Another ball rolls off the edge of the same table. Is the time taken for it to reach the ground is larger than, smaller than or the same as that of the ball is dropped from rest at the edge? Explain your answer.



the difference is

$V_{0x} \neq 0$

still

$V_{0y} = 0$

since $V_{0y} = 0$ the previous calculation will give the same result as above

(+3)

/10

Question 3

An ant walks with a constant speed counterclockwise around the inside of a circular loop of wire. Which of the following (choose one) regarding the ant's instantaneous acceleration, $\vec{a}$, at point A is true?

i) $\vec{a} = 0$.

ii) $\vec{a} \neq 0$ and is $\uparrow$.

iii) $\vec{a} \neq 0$ and is $\downarrow$.

iv) $\vec{a} \neq 0$ and is $\rightarrow$.

uniform circular motion

$\Rightarrow$ a radially inward.

$\uparrow$

$a = \frac{v^2}{r} > 0$

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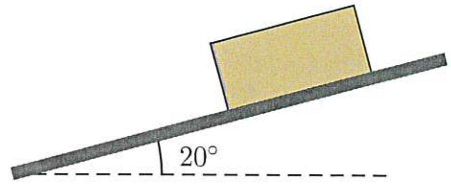
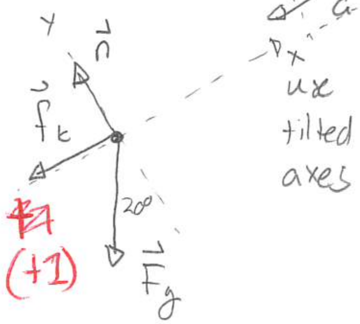
Question 4

A 30 kg box can move along the illustrated inclined ramp. It approaches the ramp from the left and slides up the ramp. The coefficient of kinetic friction between the box and the ramp is 0.50.



a) Determine the magnitude of the acceleration of the box.

$$\sum F_x = ma_x = ma$$
$$\sum F_y = ma_y = 0$$
(+1)



(+1)

	x	y
$\vec{F}_g$	$-mg\sin 20°$	$-mg\cos 20°$
$\vec{n}$	0	n
$\vec{f}_k$	$-f_k$	0

(+2)

$F_g = mg$ (+1)

$f_k = \mu_k n$ (+1)

$$\sum F_y = 0 \Rightarrow -mg\cos 20° + n = 0 \Rightarrow n = mg\cos 20° \quad (+2)$$

$$\sum F_x = ma \Rightarrow \underbrace{-mg\sin 20° - f_k = ma}_{} \Rightarrow -mg\sin 20° - \mu_k n = ma$$

(~~+3~~) (+3)

$$\Rightarrow -\cancel{m}g\sin 20° - \mu_k \cancel{m}g\cos 20° = \cancel{m}a$$

$$\Rightarrow a = -g(\sin 20° + \mu_k\cos 20°)$$
$$= -9.80\,m/s^2(\sin 20° + 0.50\cos 20°)$$
$$= -8.0\,m/s^2$$

(+3)

b) The entire situation is repeated with a box with lower mass. Will the acceleration be different to that for the ~~50~~ 30 kg box? Explain your answer.

It will be the same. The mass cancels in the derivation of part a)

(~~+3~~) (+2)

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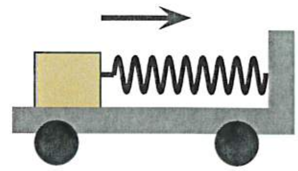
Question 5

A box lies on the frictionless surface of a cart and is anchored to one end of the cart by a spring. The cart moves horizontally and the bed of the cart stays horizontal. The cart, box and spring all move with constant speed to the right. Which of the following (choose one) is true regarding the spring?



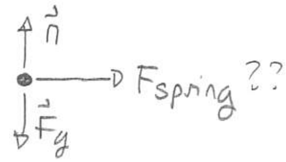
i) The spring is relaxed (neither stretched or compressed from its equilibrium position).

ii) The spring is compressed.

iii) The spring is stretched.

iv) Whether the spring is compressed or stretched depends on the mass and speed of the block.

Briefly explain your answer.



$\sum F_x = ma_x = 0$ since speed constant

$\Rightarrow F_{spring} = 0$

$\Rightarrow$ spring is neither stretched nor compressed

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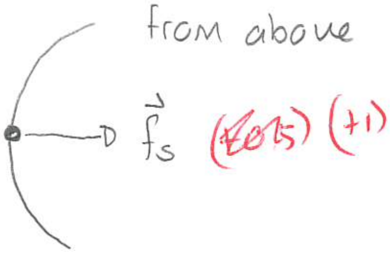
Question 6

A 2000 kg car travels along a horizontal surface. While the car turns it follows a circular path of radius 100 m and does so at a constant speed of 20 m/s. Determine the magnitude of the frictional force that is exerted on the car (and which causes it to move in the circular path).

from above



$\sum F_x = ma_x$ (+1)

$f_s = \frac{mv^2}{r}$

$= 2000\,kg \times \frac{(20\,m/s)^2}{100\,m}$ (+3)

$\Rightarrow f_s = 8000\,N$

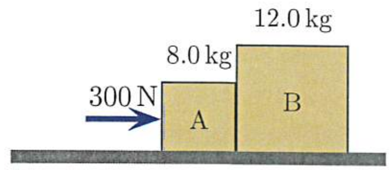
$\vec{a}$

$a = \frac{v^2}{r}$ (+1)

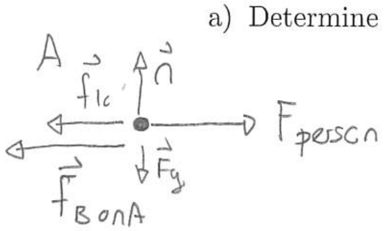
/~~8~~6

Question 7

Two crates can move along a horizontal surface. A person pushes horizontally to the right on crate A with a 300 N force. The coefficient of kinetic friction between each crate and the surface is 0.80. The two crates move in contact with each other to the right.



a) Determine the acceleration of each crate.



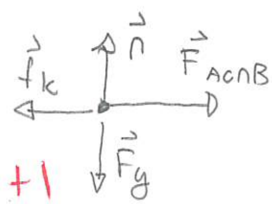
+1

magnitude of F_{BonA}

$$\sum F_x = m_A a \Rightarrow F_{person} - f_k - F_{BonA} = m_A a$$

$$\sum F_y = m_A 0 \Rightarrow n = m_A g$$

Then $f_k = \mu_k n \Rightarrow \boxed{F_{person} - \mu_k m_A g - F_{BonA} = m_A a}$



+1

$$\sum F_x = m_B a \Rightarrow F_{AonB} - \mu_k n = m_B a$$

$$\sum F_y = m_B 0 \Rightarrow n = m_B g$$

$$\Rightarrow \boxed{F_{AonB} = \mu_k m_B g + m_B a}$$

Newton's Third Law $\Rightarrow F_{A on B} = F_{B on A}$ so combining gives

$$F_{person} - \mu_k m_A g - \mu_k m_B g - m_B a = m_A a \Rightarrow (m_A + m_B) a = F_p - \mu_k g (m_A + m_B)$$

$$\Rightarrow a = \frac{F_p}{m_A + m_B} - \mu_k g = \frac{300\,N}{20\,kg} - 0.80 \times 9.8\,m/s^2 = 7.2\,m/s^2$$

b) Determine the force that crate A exerts on crate B.

+4

$$F_{AonB} = (\mu_k g + a) m_B = (0.8 \times 9.8\,m/s^2 + 7.2\,m/s^2)\, 12.0\,kg$$

$$= 180\,N$$

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Question 8

below

Two balls are fired with the same speeds from a bridge above a lake. Ball A is fired horizontally and ball B is is fired ~~downward~~ at an angle of 30° ~~from~~ the horizontal. Both eventually hit the lake. Which of the following (choose one) is true at the instant just before the balls hit the lake? Ignore air resistance.

$E_f = E_i$

$K_f = U_{gi}$ ← same

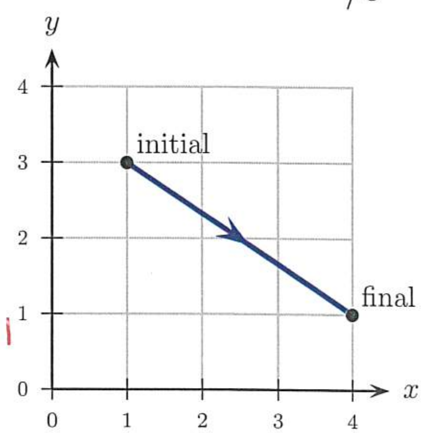
same ⇒ same v

i) The speed of ball A is the same as that of B.
ii) The speed of ball A is smaller than that of B.
iii) The speed of ball A is larger than that of B.

/5

Question 9

A ball moves along the illustrated path. Throughout the motion a hidden object exerts a force $\vec{\mathbf{F}} = 4\,\mathrm{N}\hat{\mathbf{i}} + 4\,\mathrm{N}\hat{\mathbf{j}}$ on the ball. Determine the work done by this force on the ball as it moves from the indicated initial to final location. The distances on the graph are in meters.



+1 $\left[\; W = \vec{F} \cdot \Delta \vec{r} \right.$ $\quad \Delta \vec{r} = +3m\hat{i} - 2m\hat{j}$ +1

$$\left.\begin{aligned} W &= (4N\hat{i} + 4N\hat{j})(+3m\hat{i} - 2m\hat{j}) \\ &= +12\,N\cdot m - 8\,N\cdot m \\ &= 4\,N\cdot m \end{aligned}\right\} +4$$

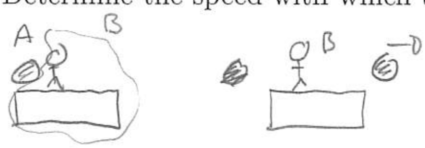
/6

Question 10

A person and sled (combined mass 100 kg) move along a horizontal icy surface with speed 2.0 m/s. The person holds a 20 kg rock which initially moves moves along with the sled. The person subsequently throws the rock horizontally and after this the person and sled are at rest. Determine the speed with which the rock must be thrown for this to happen.

A = rock

B = person/sled



No net external force ⇒ $\vec{p}$ constant

$p_f = p_i$

$m_A v_{Af} + \cancel{m_B v_{Bf}} = m_A v_{Ai} + m_B v_{Bi}$

$v_{Ai} = 2.0\,m/s$ $\quad v_{Af} = ?$

$v_{Bi} = 2.0\,m/s$ $\quad v_{Bf} = 0\,m/s$

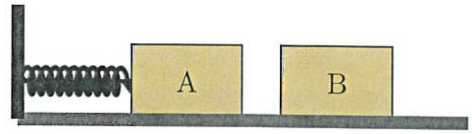
$20kg\; v_{Af} = (100kg + 20kg)\, 2.0\,m/s$

$\Rightarrow v_{Af} = \frac{120kg}{20kg} 2.0\,m/s = 12\,m/s$

/6

Question 11

Two 5.0 kg blocks are initially at rest on a horizontal frictionless surface. Block A is held against a spring, whose spring constant is 5000 N/m, compressing it by 0.10 m. Block A is released, and leaves the spring before colliding with block B. After the collision, the two blocks stick together. Determine the speed of the blocks after they collide.



Three moments

Release   Just before collision   Just after

energy conserved   momentum conserved

$V_{A0} = 0\,m/s$ $V_{A1} = ?$ $V_{B1} = 0$ $V_{A2} = V_{B2} = V_2$

$\Delta S_0 = 0.10\,m$ $\Delta S_1 = 0\,m$

10

from <u>release to just before</u>

$$E_1 = E_0 \Rightarrow K_1 + \cancel{U_{sp1}} = \cancel{K_0} + U_{sp0}$$

$$\Rightarrow \tfrac{1}{2} m_A V_{A1}^2 = \tfrac{1}{2} k (\Delta S_0)^2$$

$$\Rightarrow V_{A1} = \sqrt{\frac{k(\Delta S_0)^2}{m}} = 3.2\,m/s$$

6

From just before to just after

$$\vec{p}_2 = \vec{p}_1 \Rightarrow m_A V_{A2} + m_B V_{B2} = m_A V_{A1} + \cancel{m_B V_{B1}}$$

$$\Rightarrow (m_A + m_B) V_2 = m_A V_{A1}$$

$$\Rightarrow V_2 = \frac{m_A}{m_A + m_B} V_{A1} = \frac{1}{2} V_{A1}$$

$$= 1.6\,m/s$$

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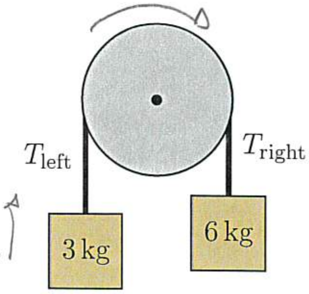
Question 12

Two blocks are connected by a massless string that runs over a 4 kg pulley. The pulley can rotate about a frictionless axle and, when it does this, the string does not slip. Let T_{left} be the tension in the string on the left and T_{right} the tension in the string on the right. Which of the following (choose one) is true?

accelerates



accelerates ↑ ↓ accelerate

i) $T_{\text{right}} = T_{\text{left}}$ regardless of the blocks' motion.

ii) $T_{\text{right}} > T_{\text{left}}$ regardless of the blocks' motion.

iii) $T_{\text{right}} < T_{\text{left}}$ regardless of the blocks' motion.

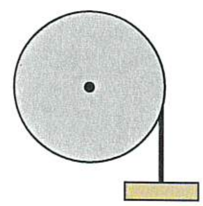
iv) Whether T_{right} is larger or smaller depends on whether the block on the right moves up or down.

For pulley $\quad \tau_{net} = I\alpha \neq 0 \Rightarrow T_{right} R > T_{left} R$

/5

Question 13

A 4.0 kg solid wheel with radius 0.10 m can rotate about a frictionless axle through its center. A 0.50 kg block is suspended from a string which is wrapped around the wheel. The system is held at rest with the block 10.0 m above the ground. The block is then released, causing the wheel to rotate (its axle does not drop). The string does not slip.



a) Describe the types of energy that the system has at the moment just before the block is released **and also** just before it hits the ground (ignore the vertical thickness of the block).

just before release – grav PE

just before ground – rotational + translational KE $\}$ 2

b) Determine the speed of the block just before it hits the ground.

$E_f = E_i$

$y_i = 1.5\,m \quad y_f = 0\,m$

$v_i = 0\,m/s \quad v_f = ?$

$\omega_i = 0\,rad/s \quad \omega_f = ?$

$K_{transf} + K_{rotf} = U_{gi}$

$\frac{1}{2} m_B v_f^2 + \frac{1}{2} I \omega_f^2 = m_B g y_i$

Then $v_f = \omega_f R$ where R is pulley radius

$\frac{1}{2} m_B v_f^2 + \frac{1}{2} I/R^2 \, v_f^2 = m_B g y_i$

$I = \frac{1}{2} m_p R^2 \Rightarrow m_B v_f^2 + \frac{1}{2} m_p v_f^2 = 2 m_B g y_i$

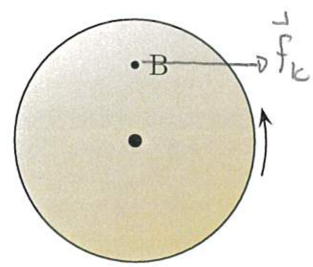
$\Rightarrow v_f^2 = 2 m_B g y_i / (m_B + \frac{1}{2} m_p) = 39.2\,m^2/s^2$

$\Rightarrow \boxed{v_f = 6.3\,m/s}$

/~~10~~ 12

Question 14 → solid, unif mass

A 40 kg disk with radius 2.0 m rotates counterclockwise about an axle through its center. At an initial moment the disk rotates with angular 40 rad/s. A brake pressing on the disk applies a constant kinetic frictional force at the point labeled B and stops the disk 10 s after the initial moment. The distance from B to the center of the disk is 1.5 m.



a) Determine the angular acceleration of the disk.

$t_0 = 0\,s$ $\quad t =$

$\omega_0 = 40\,rad/s \quad \omega = 0\,rad/s$

$\omega = \omega_0 + \alpha \Delta t$ (+1)

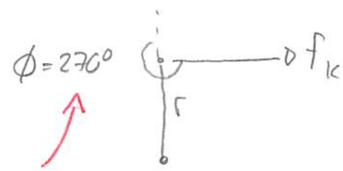
$0\,rad/s = 40\,rad/s + \alpha\, 10s$

$\Rightarrow \quad \alpha = -4.0\,rad/s^2$ ~~+4~~ (+3)

(+1)

b) Determine the magnitude of the frictional force exerted by the brake on the disk.

(+1) $\tau_{net} = I\alpha$



$\tau_{net} = \tau_{axle} + \tau_{friction}$.

(+1) $\tau = rF\sin\phi \qquad r = 0$ for axle $\Rightarrow \tau_{axle} = 0$ (+1)

only one minus sign error counts

$\Rightarrow \quad \tau_{friction} = I\alpha$

(+7)

$r f_k \sin(270°) = I(-4.0\,rad/s^2)$

$-1.5m\, f_k = -I\, 4.0\,rad/s^2 \qquad I = \frac{1}{2} mR^2$ (+1)

$1.5m\, f_k = \frac{1}{2} mR^2\, 4.0\,rad/s^2$

$f_k = \frac{40kg \times (2.0m)^2}{2 \times 1.5m}\, 4.0\,rad/s^2 = 210\,N$

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Question 15

Two satellites, P and Q, orbit Earth in circles. They are the same distance from Earth. The mass of P is ten times the mass of Q. Let a_P be the magnitude of the acceleration of P and a_Q be the magnitude of the acceleration of Q. Assume that the only force acting on each satellite is Earth's gravity. Which of the following (choose one) is true?

i) $a_Q = \frac{1}{10} a_P$

ii) $a_Q = a_P$

iii) $a_Q = 10 a_P$

$$F_g = ma$$

$$G \frac{\cancel{m} M}{r^2} = \cancel{m} a$$

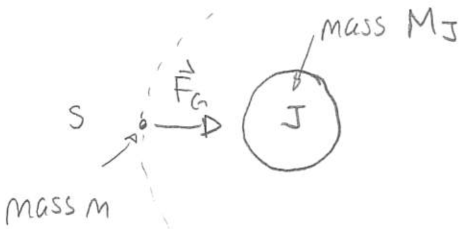
$$a = G \frac{M}{r^2}$$

/5

Question 16

A 2000 kg satellite orbits Jupiter (mass 1.9×10^{27} kg and radius 7.0×10^{7} m) in a circle. The distance from the center of Jupiter to the satellite is 2.0×10^{9} m. **Starting with and using Newton's Second Law,** derive an expression for the satellite's speed and use this to determine the time taken to complete one orbit.

with constant speed



$$\vec{F}_{net} = m\vec{a} \quad (+1)$$

$$\vec{F}_G = m\vec{a}$$

Circular motion $\Rightarrow$ $a = \frac{v^2}{r}$ (+1)

Gravity $\Rightarrow$ $F_G = G\frac{mM}{r^2}$ (+2)

$$\Rightarrow G\frac{\cancel{m}M}{r^2} = \cancel{m}\frac{v^2}{r} \Rightarrow v^2 = GM/r \Rightarrow v = \sqrt{\frac{GM}{r}} \quad (+3)$$

$$\Rightarrow v = \sqrt{\frac{6.67\times10^{-11}\,\mathrm{Nm^2/kg^2} \times 1.9\times10^{27}\,\mathrm{kg}}{2.0\times10^{9}\,\mathrm{m}}}$$

$$= 8.0\times10^{3}\,\mathrm{m/s}$$

(+1)

$$\left[v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{\Delta t} \right] \Rightarrow \Delta t = \frac{2\pi r}{v}$$

combine (+2)

$$= \frac{2\pi \times 2.0\times10^{9}\,\mathrm{m}}{8.0\times10^{3}\,\mathrm{m/s}} = 1.6\times10^{6}\,\mathrm{s}$$

/12

$= 440$ hr

$= 18$ days.

(+2)