

## Fundamental Mechanics: Class Exam 2

22 March 2018

Name: \_\_\_\_\_

Total: /70

### Instructions

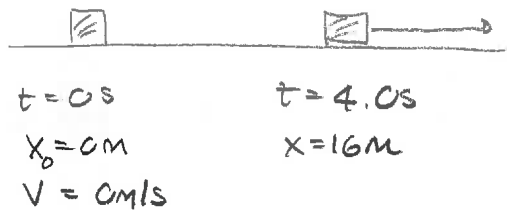
- There are 7 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

### Physical constants and useful formulae

$$g = 9.81 \text{ m/s}^2$$

### Question 1

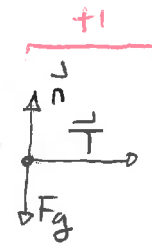
A 30 kg block can move along a horizontal frictionless surface. The block is initially at rest and is pulled with a horizontal rope, which exerts a constant force on the block for a period of 4.0 s. While this happens, the block moves 16 m. Determine the force exerted by the rope.



We need acceleration:

$$+1 \left[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \right]$$

$$+2 \left[ \begin{aligned} \Rightarrow 16 \text{ m} &= \frac{1}{2} a (4.0 \text{ s})^2 \\ \Rightarrow a &= 2.0 \text{ m/s}^2 \end{aligned} \right]$$



$$\sum F_x = ma \quad +1$$

$$\Rightarrow T = ma$$

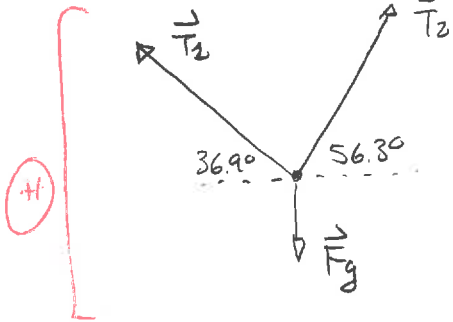
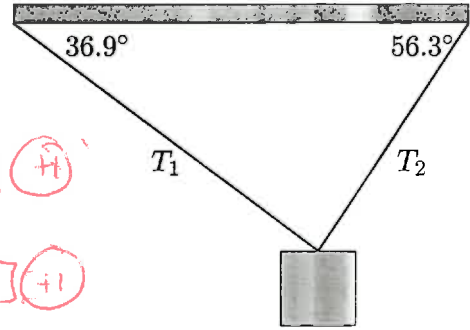
$$T = 30 \text{ kg} \times 2.0 \text{ m/s}^2$$

$$T = 60 \text{ N}$$

/8

### Question 2

A 10.0 kg block is suspended at rest by two ropes from a horizontal support in the illustrated configuration. Determine the tension in each rope.



$$\sum F_x = \text{max} = 0 \quad (+)$$

$$\sum F_y = \text{max} = 0 \quad (+)$$

$$F_g = mg = 10.0 \text{ kg} \times 9.81 \text{ m/s}^2 = 98.1 \text{ N} \quad (+)$$

	x	y
$\vec{T}_1$	$-T_1 \cos 36.9^\circ$	$T_1 \sin 36.9^\circ$
$\vec{T}_2$	$T_2 \cos 56.3^\circ$	$T_2 \sin 56.3^\circ$
$\vec{F}_g$	0	$-mg = -98.1 \text{ N}$

$$\begin{aligned} \sum F_x &= 0 \quad (+) \\ &\Rightarrow -T_1 \cos 36.9^\circ + T_2 \cos 56.3^\circ \\ &\Rightarrow -T_1 (0.800) + T_2 (0.555) = 0 \\ &\Rightarrow T_2 = T_1 \frac{0.800}{0.555} \end{aligned}$$

$$T_2 = T_1 (1.44)$$

$$\sum F_y = 0 \Rightarrow T_1 \sin 36.9^\circ + T_2 \sin 56.3^\circ - 98.1 \text{ N} = 0 \quad (+)$$

$$\Rightarrow T_1 (0.600) + T_2 (0.832) = 98.1 \text{ N}$$

$$\Rightarrow T_1 (0.600) + T_1 (1.44) (0.832) = 98.1 \text{ N}$$

$$\Rightarrow T_1 (0.600 + 1.198) = 98.1 \text{ N}$$

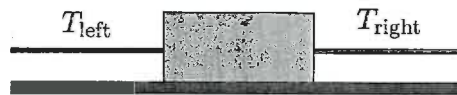
$$\Rightarrow T_1 (1.798) = 98.1 \text{ N} \Rightarrow T_1 = 54.6 \text{ N}$$

$$\text{Then } T_2 = T_1 (1.44) = 54.6 \text{ N} \times 1.44 \Rightarrow T_2 = 78.6 \text{ N}$$

/14

**Question 3**

Two ropes, pulled by people, are attached to a box that slides across a frictionless surface. After a while, the box moves right with decreasing speed. While this occurs, which of the following (choose one) is true regarding the magnitudes of the tensions in the ropes?

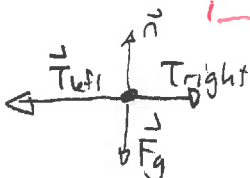


- i) The tension in the right rope must equal that in the left.
- ii) The tension in the right rope must be larger than in the left.
- iii) The tension in the left rope must be larger than in the right.

Briefly explain your answer.

If the object moves right with decreasing speed then  $\vec{a}$  is  $\leftarrow$

Thus  $\vec{F}_{net}$  is  $\leftarrow$ . This means  $T_{left} > T_{right}$



+1

+3

+2

/6

**Question 4**

Two springy rubber blocks lie on a horizontal frictionless surface as illustrated. The block on the left has a larger mass than that on the right. The blocks are compressed, then released and fly apart. During this they maintain contact with the surface. Consider only the period while the blocks “uncompress” (i.e. immediately after release but before they separate).



- a) Let  $F_{right}$  be the magnitude of the force exerted by the left block on the right block and  $F_{left}$  that by the right block on the left block. Which of the following (choose one) is true while the blocks “uncompress”?

- i)  $F_{right} < F_{left}$ .
- ii)  $F_{right} > F_{left}$ .
- iii)  $F_{right} = F_{left}$ .

Newton's Third law  $\Rightarrow$  forces same magnitude.

+4

- b) Let  $a_{right}$  be the magnitude of the acceleration of the right block  $a_{left}$  that of the left block. Which of the following (choose one) is true while the blocks “uncompress”?

- i)  $a_{right} < a_{left}$ .
- ii)  $a_{right} > a_{left}$ .
- iii)  $a_{right} = a_{left}$ .

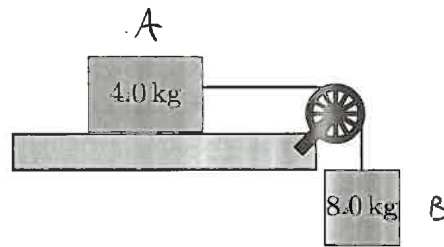
$a_{right} = \frac{F_{right}}{m_{right}}$  & larger  
 $\Rightarrow a_{right}$  smaller

+4

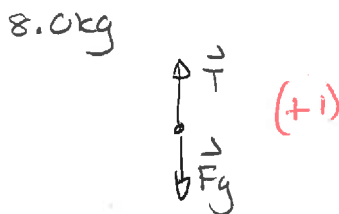
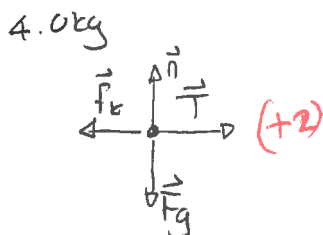
/8

### Question 5

A 4.0 kg wooden block lies on a horizontal rough surface. The coefficient of kinetic friction between the block and the surface is 0.60. The block is connected by a massless string, that runs horizontally over a massless pulley, to another suspended 8.0 kg block. Both blocks are initially at rest and then released and move.



- a) Draw a free body diagram for each of the blocks while they move.



- b) Determine the magnitude of the acceleration of the blocks while they move.

4.0 kg block:

$$\begin{aligned} (+1) \sum F_x = M_A a &\Rightarrow T - f_k = M_A a \\ (+1) \sum F_y = M_A a_y = 0 &\Rightarrow n - F_g = 0 \\ &\Rightarrow n = M_A g \quad (+1) \end{aligned}$$

Then  $f_k = \mu_k n$  gives  $T - \mu_k n = M_A a$

$$\Rightarrow T - \mu_k M_A g = M_A a \quad (+1)$$

8.0 kg block

$$\sum F_y = M_B a_y \Rightarrow T - M_B g = M_B (-a) \quad a_y = -a \quad (+2)$$

$$\Rightarrow T = M_B g - M_B a$$

Substitute:  $M_B g - M_B a - \mu_k M_A g = M_A a$

$$\Rightarrow (M_B - \mu_k M_A) g = (M_A + M_B) a$$

Question 5 continued ...

$$\begin{aligned} \text{Thus } a &= \frac{m_B - \mu_k m_A}{m_A + m_B} g \\ &= \frac{8.0 \text{ kg} - 0.60 \times 4.0 \text{ kg}}{12.0 \text{ kg}} 9.81 \text{ m/s}^2 \end{aligned}$$

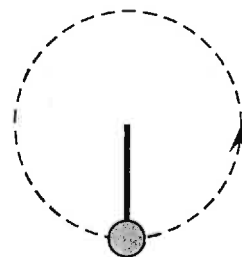
$$a = 4.58 \text{ m/s}^2$$

/16

### Question 6

A ball with mass  $m$  swings at the end of a string in a vertical circle. Which of the following (choose one) is true regarding the tension,  $T$ , in the string when the ball is at the *lowest point* in the circle?

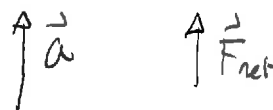
- i)  $T = mg$  regardless of speed.
- ii)  $T > mg$  regardless of speed.
- iii)  $T < mg$  regardless of speed.
- iv)  $T > mg$  when the speed is large enough and  $T < mg$  when the speed is small enough.



Briefly explain your choice.

The acceleration is radially inward

Thus net force is radially inward



$$\Rightarrow T > F_g$$

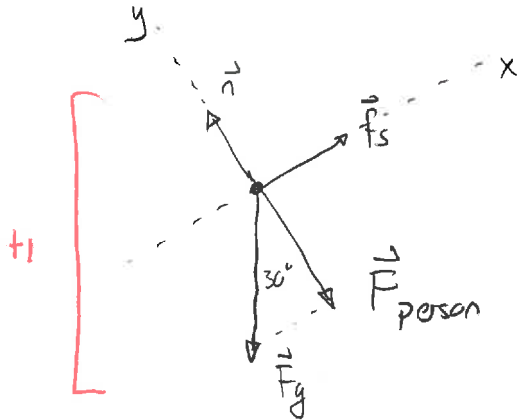
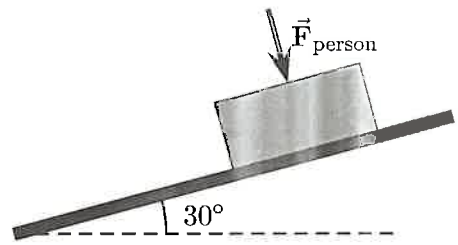
$$\Rightarrow T > mg$$

This is independent of speed

/6

### Question 7

A 50 kg box is on an inclined ramp. The coefficient of kinetic friction between the block and ramp is 0.30 and the coefficient of static friction is 0.40. A person pushes perpendicular to the surface and the block stays at rest; without this force the block would slip down the ramp. Determine the minimum force that the person must exert to keep the block at rest.



At rest  $\Rightarrow \vec{a} = 0$

$$\begin{aligned} \Rightarrow \sum F_x = Ma_x = 0 \\ \sum F_y = Ma_y = 0 \end{aligned} \quad ] +1$$

$$F_y = mg$$

$$\text{max friction} \Rightarrow f_s = \mu_s n \quad ] +1$$

	x	y
$\vec{n}$	0	n
$\vec{F}_{\text{person}}$	0	$-F_{\text{person}}$
$\vec{f}_s$	$\mu_s n$	0
$\vec{F}_g$	$-mg \sin 30^\circ$	$-mg \cos 30^\circ$

] +1

] +2

$$\sum F_x = 0 \Rightarrow \underbrace{\mu_s n - mg \sin 30^\circ = 0}_{+2} \Rightarrow n = \frac{mg \sin 30^\circ}{\mu_s} = \frac{50 \text{ kg} \times 9.81 \text{ m/s}^2 \sin 30^\circ}{0.40}$$

$$n = 613 \text{ N} \quad ] +1$$

$$\sum F_y = 0 \Rightarrow \underbrace{n - F_{\text{person}} - mg \cos 30^\circ = 0}_{+2}$$

$$\Rightarrow n - mg \cos 30^\circ = F_{\text{person}}$$

$$\begin{aligned} \Rightarrow F_{\text{person}} &= 613 \text{ N} - 50 \text{ kg} \times 9.81 \text{ m/s}^2 \cos 30^\circ \\ &= 188 \text{ N} \end{aligned} \quad ] +1 \quad /12$$