Fundamental Mechanics: Class Exam I

15 February 2018

Name: Solution **Total:** /70

Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$g = 9.81\ \mathrm{m/s^2}$

Question 1

A cart slides on a horizontal surface. At an initial instant it moves with speed 10 m/s to the right. Immediately after this instant it slows (with constant acceleration) to a stop in 5.0 s.

a) Determine the acceleration of the cart while it slows to a stop.

$10\,m/s$ → $0\,m/s$ → $t = 5.0\,s$

$x_0 = 0$ $x = ?$

$v_0 = 10\,m/s$ $v = 0\,m/s$

$$a = \frac{\Delta v}{\Delta t} = \frac{0\,m/s - 10\,m/s}{5.0\,s}$$ +1

$$a = -2.0\,m/s^2$$ +1

+2

b) Determine the distance traveled by the cart while it slows to a stop.

$$v^2 = v_0^2 + 2a(x - x_0)$$ +1

$$(0\,m/s)^2 = (10\,m/s)^2 + 2(-2.0\,m/s^2)(x - 0)$$ +2

$$\Rightarrow -100\,m^2/s^2 = -4.0\,m/s^2\, x$$ +1

$$\Rightarrow x = 25\,m$$

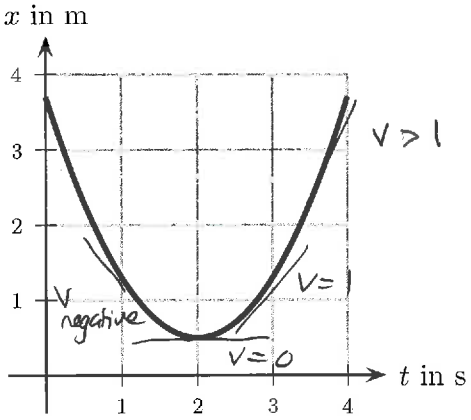
+3

/8

Question 2

An ant walks along a straight stick. The graph illustrates the ant's position vs time. During the period from 0 s to 4 s is the acceleration of the ant negative, positive or zero?

Explain your answer.



The acceleration is rate of change of velocity. Velocity is slope of position vs time. This means velocity does:

+2 [negative → 0 → positive (increasing) ⇒ positive change

⇒ Acceleration is positive !

+2

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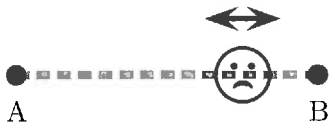
Question 3

An atom bounces back and forth along a straight line between the points labeled A and B. As it approaches A it slows, then reverses direction and speeds up toward B. As it approaches B it slows, then reverses direction and speeds up toward A. In the following, rightward motion is positive.



a) Which of the following (choose one) is true of the atom's acceleration?

+4

i) Positive as it reaches A, positive as it leaves A.
ii) Positive as it reaches A, negative as it leaves A.
iii) Negative as it reaches A, positive as it leaves A.
iv) Negative as it reaches A, negative as it leaves A.

Before reaching: v_0 ← , v ←

⇒ Δv is positive

After leaving v goes from 0 to pos ⇒ Δv pos

⇒ a > 0 both

b) Which of the following (choose one) is true of the atom's acceleration?

+4

i) Positive as it reaches B, positive as it leaves B.
ii) Positive as it reaches B, negative as it leaves B.
iii) Negative as it reaches B, positive as it leaves B.
iv) Negative as it reaches B, negative as it leaves B.

Approaching B: earlier 2 m/s → , later 1 m/s → ⇒ Δv = −1 m/s negative

⇒ a negative

After leaving B: earlier ← −1 m/s, later ← −2 m/s ⇒ Δv = −1 m/s

⇒ a negative

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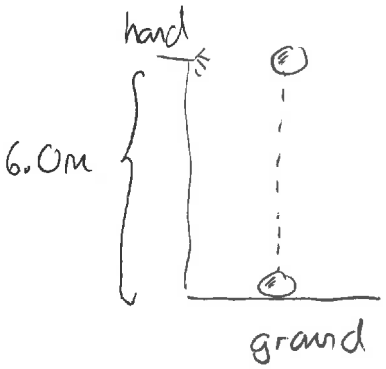
Question 4

A ball is launched vertically and leaves the ground with speed 12.0 m/s. It travels upward toward a person at the top of a building that is 6.0 m high. The person unsuccessfully attempts to catch it as it passes upward (he does not touch the ball) but manages to catch it as it passes him while it falls back down to the ground.

a) Determine the ball's speed as it passes the person while it moves upward.



$t=?$

$y_0 = 0m \quad y = 6.0m$] +1

$v_{0y} = 12.0 m/s$] +1 $v_y = ?$

$a_y = -9.8 m/s^2$

+1 $\left[v_y^2 = v_{0y}^2 + 2a_y(y-y_0) \right.$

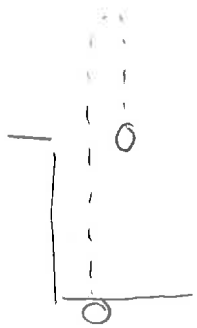
$\Rightarrow v_y^2 = (12.0 m/s)^2 - 2(9.8 m/s^2)6.0m$

$= 26.4 m^2/s^2$

$\Rightarrow v_y = \sqrt{26.4\ m^2/s^2}$

$v_y = 5.1 m/s$ +3

b) Determine the time that passes between the moment that the ball is launched and when it is caught.



same data as above

or equivalent other...

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$] +1

$\Rightarrow 6.0m = 0m + 12.0 m/s\, t - \frac{1}{2}(9.8 m/s^2)\, t^2$

$\Rightarrow 0 = -6.0m + 12.0 m/s\, t - 4.9 m/s^2 t^2$

$\Rightarrow 4.9 m/s^2 t^2 - 12.0 m/s\, t + 6.0m = 0$ +5

$$t = \frac{-(-12.0 m/s) \pm \sqrt{(-12.0 m/s)^2 - 4 \cdot 4.9 m/s^2 \cdot 6.0m}}{2 \times 4.9 m/s^2}$$

$$= \frac{12.0 m/s \pm 5.1 m/s}{9.8 m/s^2}$$

later time is when it passes on way down $\Rightarrow t = 1.74 s$

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$\vec{v}_0$ →

Question 5

A red ball and a blue ball are launched horizontally from the same height above a horizontal floor. The red ball is launched with twice the speed of the blue ball. Ignoring air resistance, which of the following (choose one) is true?

i) Both balls hit the floor at the same time.
ii) The time taken for red ball to hit the floor is exactly half the time taken for the blue ball to hit the floor.
iii) The time taken for red ball to hit the floor is less than half the time taken for the blue ball to hit the floor.
iv) The red ball hits the floor before the blue ball but takes more than half the time taken for the blue ball to hit the floor.
v) The blue ball hits the floor before the red ball.

Vertical component of motion independent of horizontal

In both cases $v_{0y} = 0$

In both cases y, y_0 same

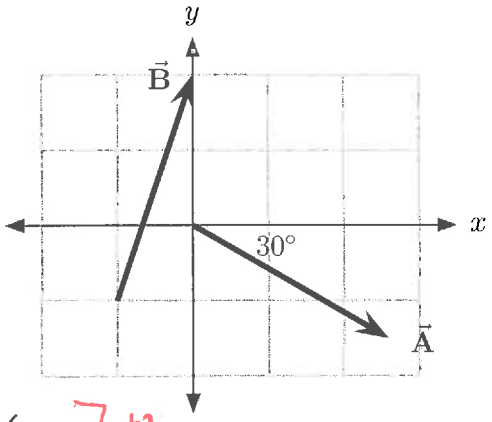
⇒ same time

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \quad \Rightarrow t = \sqrt{\frac{2(y_0 - y)}{g}}$

/5

Question 6

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, are illustrated. The grid units are in meters. The magnitude of $\vec{\mathbf{A}}$ is 3.0 m and the angle between it and the horizontal axis is 30°. Determine the magnitude of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.



$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$A_x = A\cos 30° = 3.0\text{m}\cos 30° = 2.6\text{m}$] +2

$A_y = -A\sin 30° = 3.0\text{m}\sin 30° = -1.5\text{m}$] +2

↳ -sign incorrect (-2)

$\vec{A} = 2.6\hat{i} - 1.5\hat{j}$ (in m)

$\vec{B} = \hat{i} + 3\hat{j}$

+1 +1

$\vec{C} = 3.6i + 1.5\hat{j}$] +2

$C = \sqrt{(3.6)^2 + (1.5)^2}$] +2

$C = 3.9$

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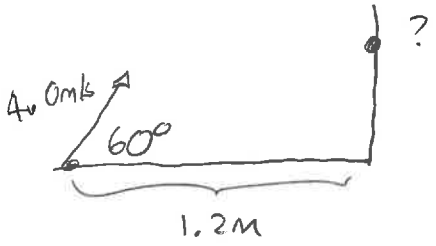
Question 7

A bug sits on a horizontal floor and launches itself with a speed of 4.0 m/s at an angle of 60° above the horizontal. It lands on the vertical side of a box which is a horizontal distance of 1.2 m from when it launched.

a) Determine the height (above the floor) of the location on the box where the bug lands.



$t = ?$

$x_0 = 0m$ $y_0 = 0m$

$x = 1.2m$ $y = ?$

$V_{ox} =$ $V_x =$

$V_{oy} =$ $V_y =$

$a_x = 0 m/s^2$ $a_y = -9.8 m/s^2$ +1

$V_{ox} = V_o \cos 60°$

$= 4.0 m/s \cos 60°$

$= 2.0 m/s$ +2

$V_{oy} = V_o \sin 60°$

$= 4.0 m/s \sin 60° = 3.46 m/s$ +2

↳ $y = y_0 + V_{oy} t + \frac{1}{2} a_y t^2$ +1

$= 0m + 3.46 m/s\, t - 4.9 m/s^2\, t^2$

Need t.

$x = x_0 + V_{ox} t + \frac{1}{2} a_x t^2$ +1

$\Rightarrow 1.2m = 2.0 m/s\, t \Rightarrow t = 0.60s$ +1

Substitute $y = 3.46 m/s \times 0.60s - 4.9 m/s^2 (0.60s)^2$ +1

$= 2.08m - 1.76m$

$y = 0.32m$

Question 7 continued ...

b) Does the bug pass the highest point of its trajectory before it lands on the box (i.e. is returning back to the floor or is still rising when it lands on the box)? Explain your answer.

When does it reach highest point? → or some equivalent...

At highest point $v_y = 0$. So

+5

$$v_y = v_{oy} + a_y t$$

$$\Rightarrow 0\,m/s = 3.46\,m/s - 9.8\,m/s^2\, t$$

$$\Rightarrow t = \frac{-3.46\,m/s}{-9.8\,m/s^2} = 0.35s$$

This occurs before it hits box

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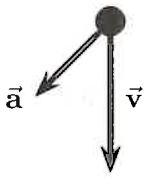
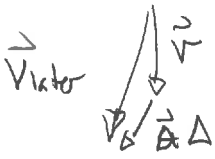
Question 8

A ball moves on a flat horizontal surface. Viewed from above, the velocity and acceleration at one instant are as illustrated. Describe whether the ball's speed increases, decreases or stay constant in the moments that follow. Does the ball's trajectory curve left, curve right or remain straight in the moments that follow? Explain you answers.

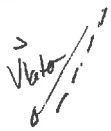


$$\vec{V}_{later} = \vec{V} + \vec{a}\,\Delta t$$



$\Rightarrow$ V_{later} is larger than $\Rightarrow$ speed increases

$\Rightarrow$ trajectory is angled



$\Rightarrow$ curves left

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