Fundamental Mechanics: Final Exam (Vesion 2)

7 May 2018

Name: Solution **Total:** /150

Instructions

- There are 17 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

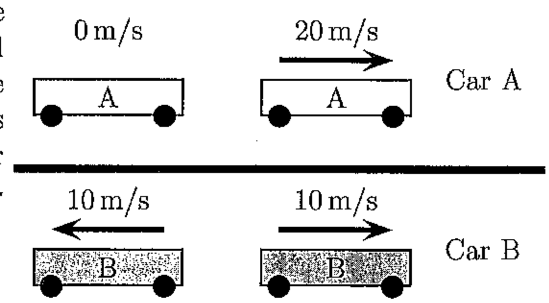
Physical constants and useful formulae

$g = 9.80\,\mathrm{m/s^2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

Question 1

Two cars each travel along a horizontal surface. The speed and direction of travel of each car is indicated at the same initial instant (left of diagram) and at the same final instant (right of diagram). The diagram is NOT to scale. Is the acceleration of car A smaller than, larger than or the same as that of car B? **Explain your answer**



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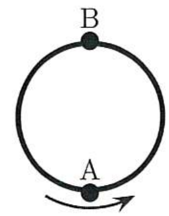
Question 2

A bug walks at a constant speed around the inside of a loop which is oriented vertically. Point A is at the bottom of the loop and point B is at the top. Which of the following (choose one) is true regarding the directions of the acceleration, $\vec{a}$ of the bug at the two illustrated points?



i) $\vec{a}$ is $\uparrow$ at A; $\vec{a}$ is $\uparrow$ at B.

ii) $\vec{a}$ is $\uparrow$ at A; $\vec{a}$ is $\downarrow$ at B.

iii) $\vec{a}$ is $\downarrow$ at A; $\vec{a}$ is $\uparrow$ at B.

iv) $\vec{a}$ is $\downarrow$ at A; $\vec{a}$ is $\downarrow$ at B.

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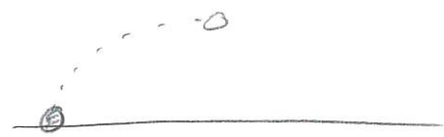
Question 3

A ball is thrown from the ground with speed 25.0 m/s at an angle of 30° above horizontal. Ignore air resistance in this problem.

a) Determine the time taken for the ball to reach its highest point above ground.

+4
$V_{ix} = V_i \cos\theta = 25.0 m/s \cos 30° = 22 m/s$
$V_{iy} = v_i \sin\theta = 25.0 m/s \sin 30° = 12.5 m/s$



$x_i = 0m$ $x_f =$
$y_i = 0m$ $y_f =$
$V_{ix} =$ $V_{fx} =$
$V_{iy} = 12.5 m/s$ $v_{fy} = 0 m/s$
$a_x = 0 m/s^2$ $a_y = -9.8 m/s^2$

+1 $V_{fy} = V_{iy} + a_y \Delta t$

+1 +1
$0 m/s = 12.5 m/s - 9.8 m/s^2 \Delta t$

$\Rightarrow$ +1 $\Delta t = \frac{12.5 m/s}{9.8 m/s^2} = 1.3 s$

b) Determine the speed of the ball at its highest point above ground.

+1
$V_{fx} = V_{ix} + \cancel{a_x} \Delta t = 22 m/s$
$V_{fy} = 0 m/s$
+1
$\Rightarrow$ +2 $V_f = 22 m/s$

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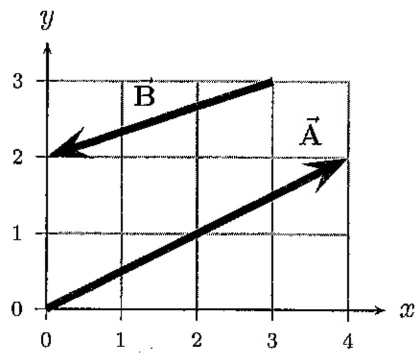
Question 4

Two vectors are as illustrated. Express $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ in terms of standard unit vectors ($\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$).



$\vec{A} = 4\hat{i} + 2\hat{j}$

$\vec{B} = -3\hat{i} - \hat{j}$

$\vec{C} = \vec{A} + \vec{B} = 4\hat{i} + 2\hat{j} - 3\hat{i} - \hat{j}$

$\vec{C} = \hat{i} + \hat{j}$

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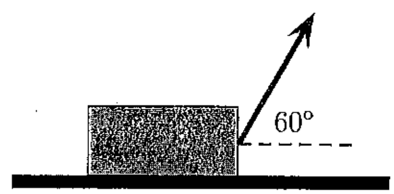
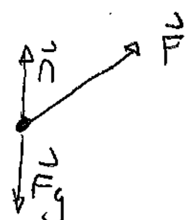
Question 5

A rope pulls a block, with mass m, along a horizontal frictionless surface. The block moves horizontally while the rope pulls at the illustrated angle. Which of the following (choose one) is true regarding the magnitude of the normal force, n, exerted by the floor on the block?



i) $n = mg$
ii) $n > mg$
iii) $n < mg$



$\sum F_y = ma_y = 0$

$n + F\sin 60° - mg = 0$

$n = mg - F\sin 60°$

$n < mg$

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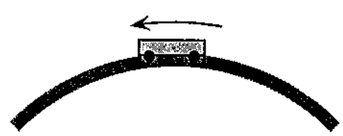
Question 6

A cart of mass m moves over a bump, with a circular cross-section, above Earths surface as illustrated. The cart's speed at the top of the bump is non-zero. Which of the following (choose one) is true regarding the magnitude of the normal force, n, exerted by the loop on the cart?



i) $n < mg$.
ii) $n = mg$.
iii) $n > mg$.

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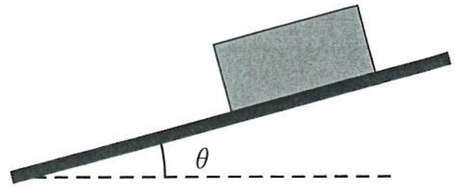
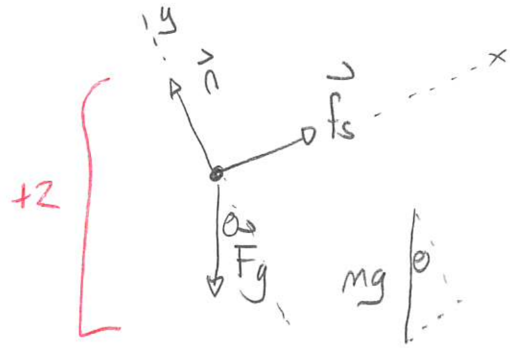
Question 7

A 40 kg box can move along a ramp which can be inclined at variable angles. The coefficient of kinetic friction between the box and the ramp is 0.20 and the coefficient of static friction is 0.30.



a) Determine the largest angle above the horizontal, θ, so that the box remains at rest on the ramp.



$$\sum F_x = ma_x = 0$$
$$\sum F_y = ma_y = 0$$ +1

$\Rightarrow f_s - mg\sin\theta = 0$

$\Rightarrow n - mg\cos\theta = 0$

	x	y
+1 $\vec{n}$	0	n
+1 $\vec{f_s}$	f_s	0
+4 $\vec{F_g}$	$-mg\sin\theta$	$-mg\cos\theta$

$f_s = mg\sin\theta$ $\qquad n = mg\cos\theta$

and $f_s < \mu_s n$ +1

+2

$\Rightarrow \cancel{m}g\sin\theta < \mu_s \cancel{mg}\cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} < \mu_s \Rightarrow \tan\theta < 0.30 \Rightarrow \theta < \tan^{-1}(0.30)$

$\Rightarrow \theta < \boxed{16.7^\circ}$

b) Suppose that the angle of the ramp is increased beyond the maximum of the previous part. Which of the following (choose one) is true?

i) The box remains at rest.

ii) The box slides down the ramp with constant speed.

iii) The box slides down the ramp with constantly increasing speed.

+4

The friction force is reduced and there is a net force down the ramp $\Rightarrow$ accelerates down.

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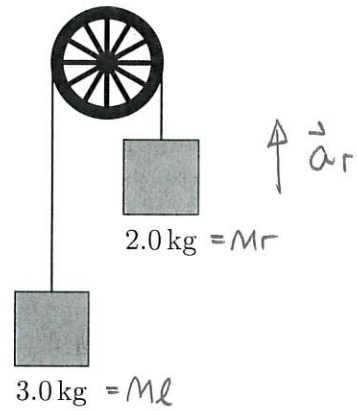
Question 8

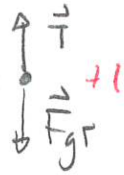
Two blocks are connected by a massless string that runs over a massless pulley. Ignore air resistance.



a) Determine the acceleration of the block on the right.

Left

Right



$\sum F_y = m_l a_{ly}$ +1 $\qquad \sum F_y = M_r a_{ry}$ +1

+2 $\Rightarrow T - m_l g = m_l a_{ly}$ $\qquad T - M_r g = M_r a_{ry}$ +2 $\Rightarrow T = M_r g + M_r a_{ry}$

$\Rightarrow T = m_l g + m_l a_{ly}$

But $a_{ly} = -a_{ry}$ +2 $\Rightarrow T = m_l g - m_l a_{ry}$

+3 Combine $M_r g + M_r a_{ry} = m_l g - m_l a_{ry} \Rightarrow (M_r + M_l)\, a_{ry} = (m_l - M_r)\, g$

$$\Rightarrow a_{ry} = \frac{m_l - m_r}{m_l + m_r}\, g = \frac{1.0\,\text{kg}}{5.0\,\text{kg}}\, 9.8\,\text{m/s}^2$$

$$= \boxed{2.0\,\text{m/s}^2}$$

b) Determine the tension in the string.

+3

$$T = M_r(g + a_{ry})$$

$$= 2.0\,\text{kg}(9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2) = \boxed{24\,\text{N}}$$

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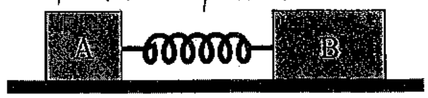
Spring exerts same force on ends

$\Rightarrow$ (i) or (ii).

Momentum conserved $\quad \overset{0}{p_{tot\,i}} = p_{tot\,f} \Rightarrow m_A v_{Af} + m_B v_{Bf} = 0$

Question 9



Two boxes on a frictionless surface are separated by a compressed spring. The mass of box B is twice the mass of box A. They are held at rest and then released. Which of the following (choose one) is true *at any instant* while the spring extends as they move apart?

i) The net force an A equals that on B. They have the same speed.
ii) The net force an A equals that on B. A has a greater speed than B.
iii) The net force an A is larger than on B. A has a greater speed than B.
iv) The net force an A is larger than on B. A has a smaller speed than B.
v) The net force an A is larger than on B. A has the same speed as B.

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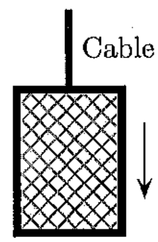
Question 10

An elevator, which is suspended by a cable, moves down.



a) The elevator slows down as it descends. Is the work done by the cable positive, negative or zero? Explain your answer.

b) The elevator speeds up as it descends. The cable does not become slack. Is the work done by the cable positive, negative or zero? Explain your answer.

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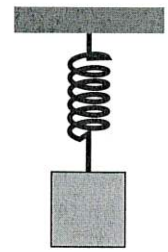
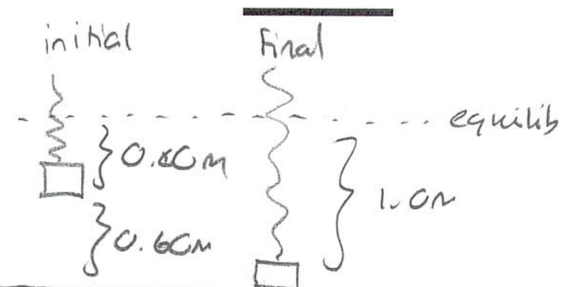
Question 11



A block is attached to a spring (spring constant 2000 N/m) that is suspended from the ceiling. The block is initially held at rest so that the spring is stretched by 0.40 m and the bottom of the block is 0.60 m above the ground The block is released and drops toward the Earth. It reverses direction at the moment that it would be about to touch the ground. Determine the mass of the block so that this occurs.

Energy is conserved



+1 $E_f = E_i$

+1 $\cancel{K_f} + \cancel{U_{gf}} + U_{spf} = \cancel{K_i} + U_{gi} + U_{spi}$

$$\underbrace{\tfrac{1}{2}k(\Delta s_f)^2}_{+2} = \underbrace{mgy_i}_{+2} + \underbrace{\tfrac{1}{2}k(\Delta s_i)^2}_{+2}$$

$v_i = 0\,m/s$ $\quad v_f = 0\,m/s$

$\underbrace{y_i = 0.60m}_{+1}$, $\underbrace{y_f = 0.0m}_{+1}$

$\underbrace{\Delta s_i = 0.40m}_{+1}$ $\quad \underbrace{\Delta s_f = 1.0m}_{+2}$

$$\Rightarrow \tfrac{1}{2}k\left[(\Delta s_f)^2 - (\Delta s_i)^2\right] = mgy_i$$

+3

$$\tfrac{1}{2}\, 2000\, N/m \left[(1.0m)^2 - (0.4m)^2\right] = m \times 9.8 m/s^2 \times 0.60m$$

$$840\,J = m \times 5.88\, m^2/s^2$$

$$\Rightarrow \quad m = \frac{840J}{5.88 m^2/s^2}$$

$$= 143kg$$

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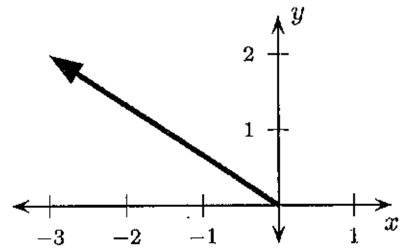
Question 12

A particle moves along the indicated straight path in the xy plane. While this happens, the following two forces act on the particle: $\vec{\mathbf{F}}_A = 2\,\text{N}\hat{\mathbf{i}} + 3\,\text{N}\hat{\mathbf{j}}$ and $\vec{\mathbf{F}}_B = -2\,\text{N}\hat{\mathbf{i}} + 3\,\text{N}\hat{\mathbf{j}}$. Let W_A be the work done by $\vec{\mathbf{F}}_A$ and W_B be the work done by $\vec{\mathbf{F}}_B$. Which of the following (choose one) is true?



i) $W_A = W_B$.

ii) $W_A > W_B$.

iii) $W_A < W_B$.

$\vec{W} = \vec{F} \cdot \Delta\vec{r}$ and $\Delta\vec{r} = -3\hat{i} + 2\hat{j}$ (in m)

$W_A = 2N \times (-3m) + 3N \times 2m = 0\,J$

$W_B = -2N \times (-3m) + 3N \times 2m = 12\,J$

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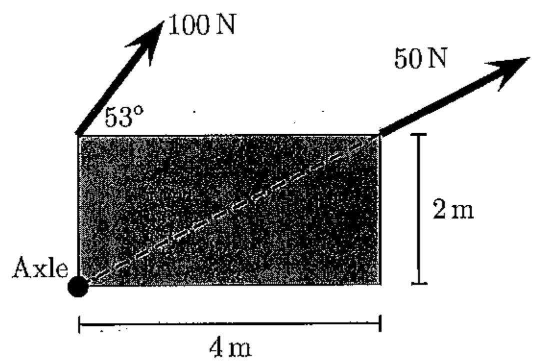
Question 13

A rectangular plate can pivot about an axle at the lower left corner and perpendicular to the page. Only two forces act on the plate, as illustrated.



a) Determine the net torque about the axle.

b) Suppose that the plate is initially rotating clockwise about the axle and the two forces then act for a while. The forces maintain their magnitudes and angles (relative to the plate). Which of the following (choose one) is true *while the forces act?*

i) The plate continues to rotate clockwise at a constant rate.

ii) The plate immediately speeds up and then rotates at a constant rate.

iii) The plate rotates at a rate which constantly increases.

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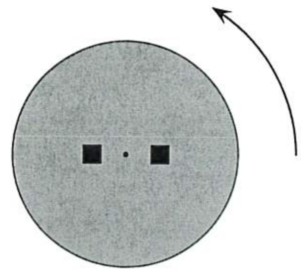
Question 14

Two 5.0 kg lead bricks are on a 8.0 kg disk with radius 0.80 m. The bricks are connected by a string and each is 0.10 m from the center of the disk. The disk and bricks all initially rotate with angular velocity 40 rad/s. At a later moment the string breaks and the bricks slide along radial grooves to the edge of the disk. Determine the angular velocity of the disk when the bricks reach the edge of the disk. The bricks can be regarded as point masses.



Angular momentum conserved

+1 $L_f = L_i$

$\underbrace{I_f}_{+1} \omega_f = I_i \omega_i$ +1

Now $I_i = \overbrace{I_{disk} + I_{bricks\,i}}^{+2}$

$= \underbrace{\frac{1}{2} M_{disk} r_{disk}^2}_{+1} + 2 \underbrace{M_{bricks} r_{bricks\,i}^2}_{+1}$

$= \frac{1}{2}\, 8.0\,kg \times (0.80\,m)^2 + 2 \times 5.0\,kg \times (\underbrace{0.10\,m}_{+1})^2$

$= 2.66\ kgm^2$

Then $I_f = \overbrace{I_{disk} + I_{bricks\,f}}^{+2}$

$= \frac{1}{2}\, 8.0\,kg \times (0.80\,m)^2 + 2 \times 5.0\,kg \times (\underbrace{0.80\,m}_{+1})^2$

$= 8.96\,kgm^2$

$\Rightarrow 8.96\,kgm^2\, \omega_f = 2.66\,kgm^2\, \omega_i$

$\Rightarrow \omega_f = \frac{2.66\,kgm^2}{8.96\,kgm^2}\, \omega_i \leftarrow 40\,rad/s$

+1

$= 12\,rad/s$

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Question 15

Two satellites, called A and B, orbit Earth in circles with constant speeds. The distance from the center of Earth to satellite B is twice that for satellite A. Which of the following (choose one) is true regarding the accelerations of the satellites?

i) Accelerations are same.

ii) Acceleration of A is double that of B.

iii) Acceleration of A is one quarter that of B.

iv) Acceleration of A is four times that of B.

$F_G = M_s a$

$G \frac{\cancel{M_s} M}{r^2} = \cancel{M_s} a$

$a = G \frac{M}{r^2}$

½ for A $\Rightarrow$ a = 4 times for A

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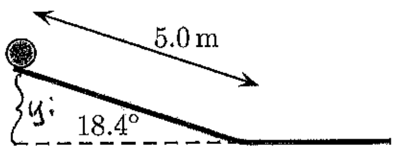
Question 16

A 0.400 kg hollow sphere with radius 0.20 m is held at rest on a 5.0 m track inclined at 18.4° above the horizontal. The sphere is released and rolls without slipping along the track. Determine the speed of the sphere at the moment that it reaches the end of the track.



$E_f = E_i$

$K_{transf} + K_{rotf} + U_{gf} = \cancel{K_{trans\,i}} + \cancel{K_{rot\,i}} + U_{gi}$

$\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + \cancel{mgy_f} = mgy_i$

$y_i = 5.0\,m \sin 18.4°$
$= 1.6\,m$

Now $I = \frac{2}{3} mR^2$ gives

$\frac{1}{2} \cancel{m} v_f^2 + \frac{1}{3} \cancel{m} (\omega_f R)^2 = \cancel{m} g y_i$ and $v_f = \omega_f R$

$\Rightarrow \left(\frac{1}{2} + \frac{1}{3}\right) v_f^2 = g y_i$

$\Rightarrow \frac{5}{6} v_f^2 = 9.8\,m/s^2 \times 1.6\,m \Rightarrow \boxed{v_f = 4.3\,m/s}$

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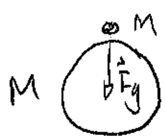
Question 17

A planet has radius 2.0×10^6 m and the acceleration due to gravity at its surface is $7.0\,\mathrm{m/s^2}$. Determine the mass of the planet. **To receive full credit, your solution must start with Newton's second law and use this to derive the answer.**

$\sum \vec{F}_{net} = m\vec{a}$



$G \frac{\cancel{m} M}{r^2} = \cancel{m} a \Rightarrow M = \frac{ar^2}{G}$

$\Rightarrow M = \frac{7.0\,m/s^2 \times (2.0 \times 10^6\,m)^2}{6.67 \times 10^{-11}\,Nm^2/kg^2}$

$= 4.2 \times 10^{23}\,kg$

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