Fundamental Mechanics: Final Exam (Version 1)

7 May 2018

Name: _____ **Total:** **/150**

Instructions

- There are 17 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

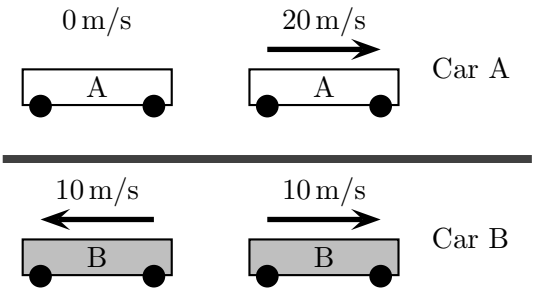
Physical constants and useful formulae

$g = 9.80\,\mathrm{m/s^2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

Question 1

Two cars each travel along a horizontal surface. The speed and direction of travel of each car is indicated at the same initial instant (left of diagram) and at the same final instant (right of diagram). The diagram is NOT to scale. Is the acceleration of car A smaller than, larger than or the same as that of car B? **Explain your answer**



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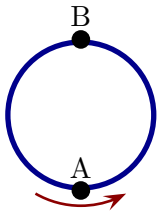
Question 2

A bug walks at a constant speed around the inside of a loop which is oriented vertically. Point A is at the bottom of the loop and point B is at the top. Which of the following (choose one) is true regarding the directions of the acceleration, $\vec{\mathbf{a}}$ of the bug at the two illustrated points?



i) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

ii) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

iii) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

iv) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

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Question 3

A bug jumps from a horizontal table with a speed of 4.0 m/s. The bug leaves the table at an angle of 60° from the table and lands back on the table. Determine the time for which the bug is in the air. Ignore air resistance in this problem.

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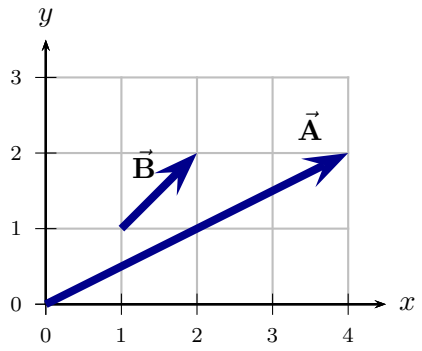
Question 4

Two vectors are as illustrated. Express $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ in terms of standard unit vectors ($\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$).



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Question 5

A phone, with mass m, is suspended in an elevator as illustrated. The elevator moves up with a decreasing speed. The rope suspending the phone has no slack throughout the motion. Which of the following (choose one) is true regarding the tension in the rope that suspends the phone?

i) $T = 0\,\text{N}$.
ii) $T = mg$.
iii) $T > mg$.
iv) $0 < T < mg$.

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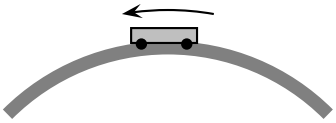
Question 6

A cart of mass m moves over a bump, with a circular cross-section, above Earths surface as illustrated. The cart's speed at the top of the bump is non-zero. Which of the following (choose one) is true regarding the magnitude of the normal force, n, exerted by the loop on the cart?



i) $n < mg$.
ii) $n = mg$.
iii) $n > mg$.

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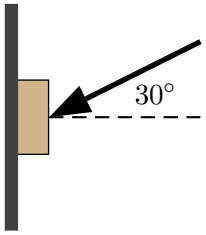
Question 7

A 1.5 kg book is pushed against a rough vertical wall by a hand which exerts a force at an angle of 30° from the horizontal. The coefficient of static friction between the book and wall is 0.80. Determine the minimum force that must be exerted by the hand to keep the book at rest.



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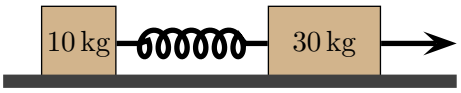
Question 8

Two boxes on a frictionless surface are separated by a spring with spring constant 200 N/m. A rope pulls the block on the right with a constant tension of 80 N. The blocks move together while a constant separation is maintained between them.



a) Determine the acceleration of the 30 kg box and the force exerted by the spring on this box.

b) Determine the force exerted by the spring on the 10 kg box.

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Question 9

A person stands at rest in a boat on a perfectly still lake; the boat is at rest (compared to the water). The person starts to walk east in the boat. Which of the following (choose one) is true while the person is walking east? Ignore air resistance and any friction between the boat and water and assume that there are no currents in the water.

i) The boat remains at rest.

ii) The boat moves east.

iii) The boat moves west.

iv) The boat moves west if the person moves fast enough. Otherwise it remains at rest.

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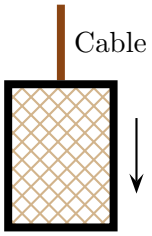
Question 10

An elevator, which is suspended by a cable, moves down.



a) The elevator slows down as it descends. Is the work done by the cable positive, negative or zero? Explain your answer.

b) The elevator speeds up as it descends. The cable does not become slack. Is the work done by the cable positive, negative or zero? Explain your answer.

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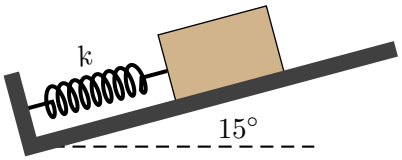
Question 11

A 2.0 kg box can move along a ramp tilted at 15° above the horizontal. It is held at rest against a spring with unknown spring constant k. The spring is compressed by 0.50 m. The box is released and it leaves the spring with speed 5.0 m/s as the spring reaches its equilibrium point. Ignoring friction and air resistance, determine k.



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Question 12

Consider the vectors,

$$\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$
$$\vec{\mathbf{B}} = 40\hat{\mathbf{i}} + 30\hat{\mathbf{j}}$$
$$\vec{\mathbf{C}} = -40\hat{\mathbf{i}} + 30\hat{\mathbf{j}}$$

Which of the following (choose one) is true?

i) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

ii) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} > \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

iii) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} < \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

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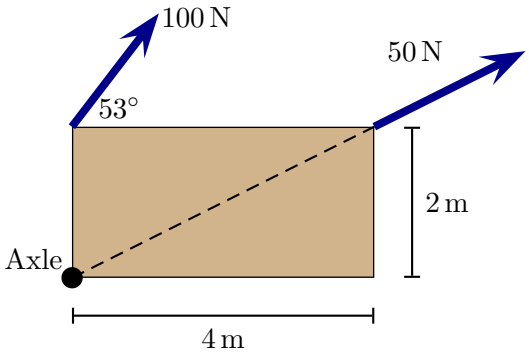
Question 13

A rectangular plate can pivot about an axle at the lower left corner and perpendicular to the page. Only two forces act on the plate, as illustrated.

a) Determine the net torque about the axle.



b) Suppose that the plate is initially rotating clockwise about the axle and the two forces then act for a while. The forces maintain their magnitudes and angles (relative to the plate). Which of the following (choose one) is true *while the forces act?*

i) The plate continues to rotate clockwise at a constant rate.

ii) The plate immediately speeds up and then rotates at a constant rate.

iii) The plate rotates at a rate which constantly increases.

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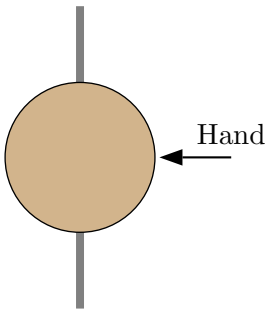
Question 14

A 2.0 kg solid sphere with radius 0.10 m rotates about an axle with negligible mass. The sphere rotates at 1800 rpm. A hand pushes with a constant force on the sphere at its midpoint and this brings the sphere to a stop in 6.0 s. Determine the magnitude of the frictional force exerted by the hand.



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Question 15

A satellite with mass m_s is a distance above the earth's surface exactly equal to the earth's radius. If the earth's radius is denoted R_E and the mass of the earth M_E, which of the following (choose one) represents the force exerted by the earth on the satellite?

i) $F = G\,\dfrac{M_E m_s}{2R_E}$

ii) $F = G\,\dfrac{M_E m_s}{R_E^2}$

iii) $F = G\,\dfrac{M_E m_s}{2R_E^2}$

iv) $F = G\,\dfrac{M_E m_s}{4R_E^2}$

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Question 16

A 2.0 kg solid sphere with radius 0.10 m rolls along a horizontal surface without slipping with a constant speed of 10 m/s. It approaches a ramp and rolls up the ramp without slipping. Determine the maximum vertical height reached by the sphere along the ramp.

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Question 17

Titan is a moon of the planet Saturn. Titan has mass 1.35×10^{23} kg and radius 2.57×10^{6} m. Determine the acceleration due to Titan's gravity at the surface of Titan. **Note: to receive full credit for this problem, your solution must with Newton's second law and use this to derive the answer.**

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